Three Pieces of Paper

John Clark

John 312

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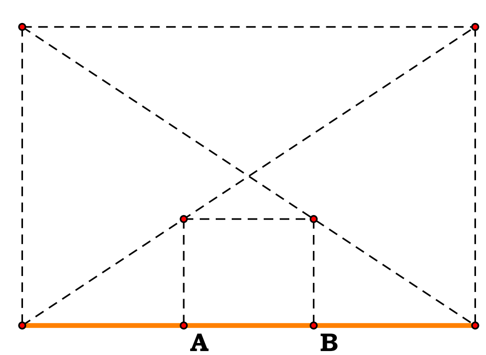
Saturday, June 17, 2023

I have been putting off this project for over twenty-two years simply because I did not believe, and rightly so, that I could be happy with the results until I had a better understanding. So, here it is, as one can divide a straight line into any number of equal segments, so too, one can divide a circle into an equal number of segments, the results are arithmetic, however, it turns out to be advanced arithmetic, a very neglected advanced form of arithmetic.

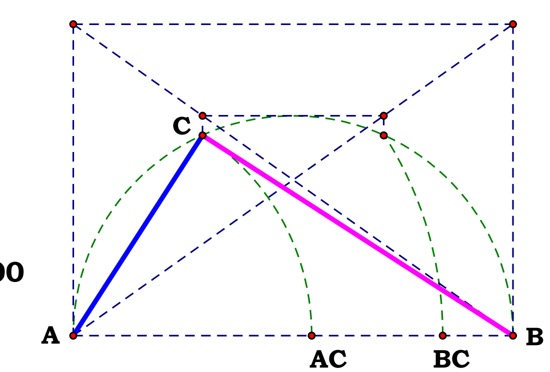
One can better understand the difference between Arithmetic Progression and Geometric Progression by way of the sum of the extremes of a progression, what does it take to achieve 1?

In terms of Arithmetic Progression, the sum is straight forward, adding result to result, and the product, if one has the extremes, simply the sum. Where a Geometric Progression, the sum of the squares are one, like so.



A = 0.35658
B = 0.64342
A + B = 1.00000

AC = 0.54219
BC = 0.84025
$AC^2 + BC^2 = 1.00000$



When it comes to a static figure, such as a circle divided an equal number of times, it will mean that the distances along the circumference are equal, but the ratio produced on the diameter is an ellipse, i.e., one ratio which will produce exactly the same angle regardless of size onto the diameter. Thus, the mathematics required, although being arithmetic, means a very complete study of the ellipse and circle.

And one will notice a difference from dynamic figures in the static figure division by angle, every name of everything in the figure resolves to arithmetic equations, numbers, and operations on them. Therefore, in writing up the division of a circle by angles, the only symbols in the whole work are arithmetic. The recursion of the unit is both arithmetic and geometric, and the operations in the one have a way of being expressed in the other without the mystic bs of contemporary and past scholars.

Archimedean Paper Trisector

Descriptions.

$AR := .75220 \quad CR := 2 - AR \quad ER := \sqrt{AR \cdot CR}$

$AE := \sqrt{AR^2 + ER^2} \quad AQ := \frac{AE}{2} \quad QS := \frac{ER}{2}$

$AS := \frac{AR}{2} \quad BS := 1 - AS \quad BQ := \sqrt{BS^2 + QS^2}$

$BT := \frac{BS}{BQ} \quad PT := \frac{QS}{BQ} \quad CT := 1 + BT \quad RU := \frac{CT \cdot ER}{PT} \quad BR := 1 - AR$

$BU := RU - BR \quad BO := \frac{1}{BU} \quad BF := BO \quad BV := \frac{BR}{BU} \quad OV := ER \cdot BO \quad CV := 1 + BV \quad EO := 1 - BO \quad NO := EO \quad NV := \sqrt{NO^2 - OV^2} \quad CN := CV - NV \quad CN - NO = 0$

$CE := 2 \cdot BQ \quad CM := CN \quad MW := \frac{ER \cdot CM}{CE} \quad MW - OV = 0$

$A = 0.00000$

$B = 1.00000$

$C = 2.00000$

$D = 2.41421$

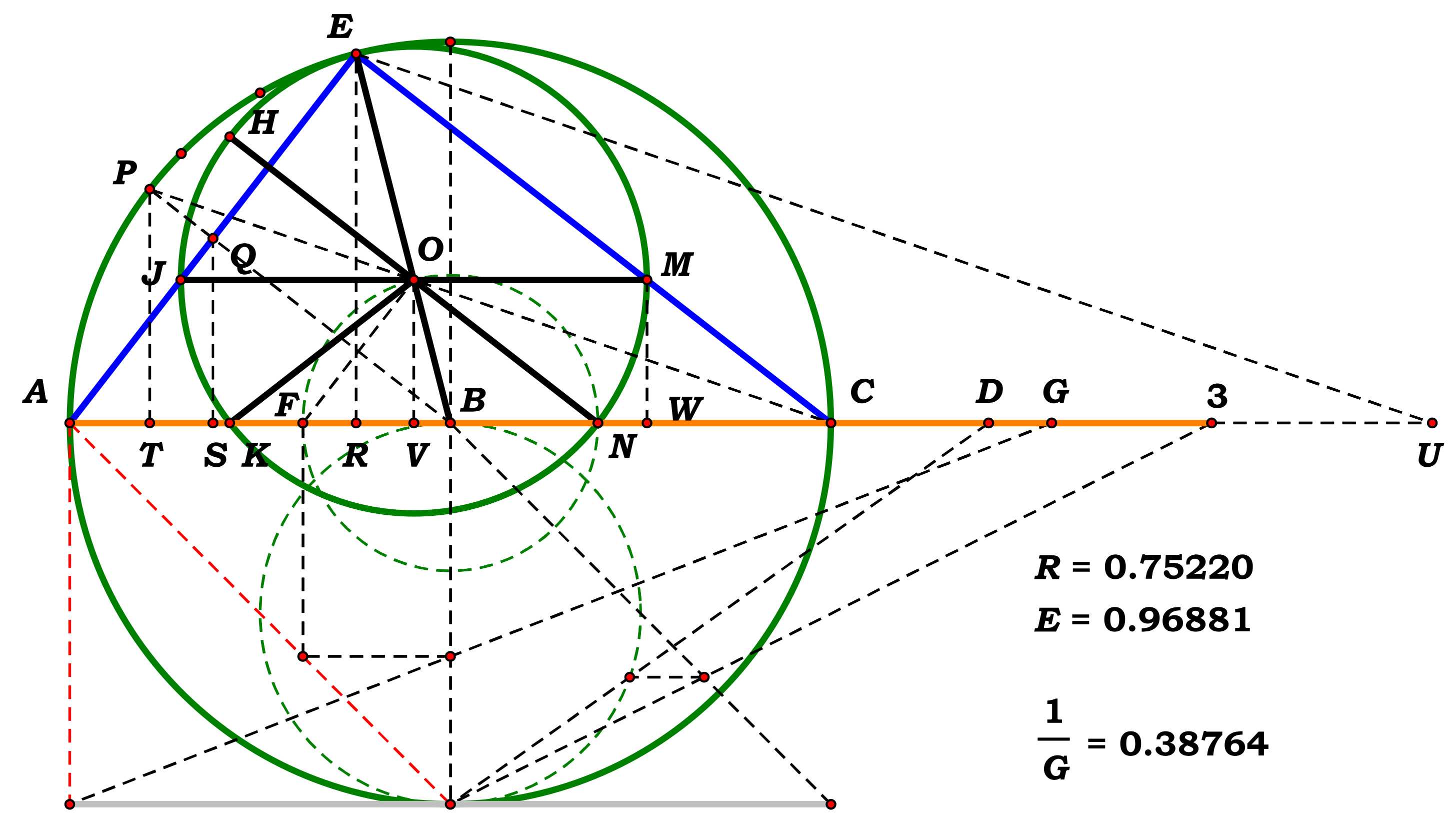
$G = 2.57974$

$G - D = 0.16553$

$m\angle EOH = 37.82643°$

$3 \cdot m\angle EOH = 113.47929°$

$\frac{3 \cdot m\angle EOH}{m\angle EOH} = 3.00000$



$R = 0.75220$

$E = 0.96881$

$\frac{1}{G} = 0.38764$

Definitions.

$CR - (2 - AR) = 0 \quad ER - \sqrt{2 \cdot AR - AR^2} = 0 \quad AE - \sqrt{2} \cdot \sqrt{AR} = 0 \quad AQ - \frac{\sqrt{2} \cdot \sqrt{AR}}{2} = 0 \quad QS - \frac{\sqrt{2 \cdot AR - AR^2}}{2} = 0 \quad AS - \frac{AR}{2} = 0 \quad BS - \frac{(2 - AR)}{2} = 0 \quad BQ - \frac{\sqrt{2 - AR}}{\sqrt{2}} = 0$

$BT - BQ = 0 \quad PT - \frac{\sqrt{2} \cdot \sqrt{2 \cdot AR - AR^2}}{2 \cdot \sqrt{2 - AR}} = 0 \quad CT - \frac{2 + \sqrt{2} \cdot \sqrt{2 - AR}}{2} = 0 \quad RU - \sqrt{2 - AR} \cdot \left(\sqrt{2 - AR} + \sqrt{2}\right) = 0 \quad BR - (1 - AR) \quad BU - \left(\sqrt{2} \cdot \sqrt{2 - AR} + 1\right)$

$BO - \frac{1}{\sqrt{2} \cdot \sqrt{2 - AR} + 1} = 0 \quad BF - BO = 0 \quad BV - \frac{(AR - 1) \cdot \left(\sqrt{2} \cdot \sqrt{2 - AR} - 1\right)}{2 \cdot AR - 3} = 0 \quad OV - \frac{\sqrt{2} \cdot \sqrt{2 \cdot AR - AR^2}}{2 \cdot \left(\sqrt{2 - AR} + \frac{\sqrt{2}}{2}\right)} = 0 \quad CV - \frac{\left(AR \cdot \sqrt{2 - AR} - \sqrt{2 - AR}\right) \cdot \sqrt{2} + AR - 2}{2 \cdot AR - 3} = 0$

$A = 0.00000$

$B = 1.00000$

$C = 2.00000$

$D = 2.41421$

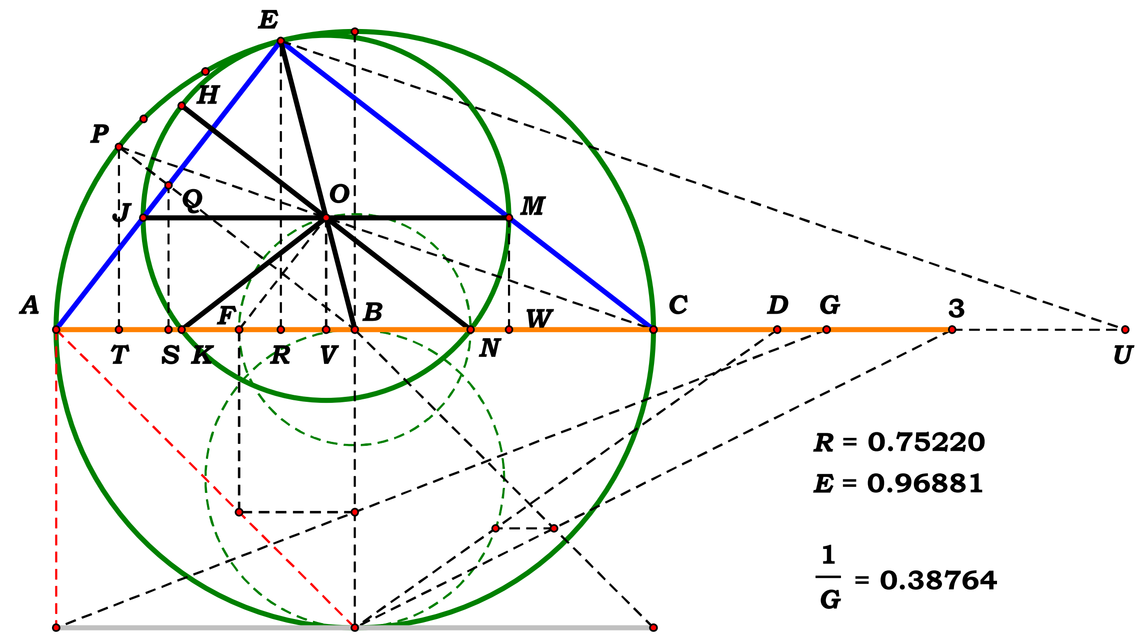
$G = 2.57974$

$G - D = 0.16553$

$m\angle EOH = 37.82643°$

$3 \cdot m\angle EOH = 113.47929°$

$\frac{3 \cdot m\angle EOH}{m\angle EOH} = 3.00000$



$R = 0.75220$

$E = 0.96881$

$\frac{1}{G} = 0.38764$

$$EO - \frac{2 \cdot AR + \sqrt{4 - 2 \cdot AR} - 4}{2 \cdot AR - 3}$$

$$NO - EO = 0$$

$$NV - \frac{(2 - AR)}{\sqrt{2 \cdot \sqrt{2} \cdot \sqrt{2 - AR} - 2 \cdot AR + 5}} = 0$$

$$CN - NO = 0$$

$$CN - \left(\frac{AR - \sqrt{2} \cdot \sqrt{2 - AR} + \sqrt{2} \cdot AR \cdot \sqrt{2 - AR} - 2}{2 \cdot AR - 3} + \frac{AR - 2}{\sqrt{2 \cdot \sqrt{2} \cdot \sqrt{2 - AR} - 2 \cdot AR + 5}} \right) = 0$$

$$CE - \sqrt{2} \cdot \sqrt{2 - AR} = 0 \qquad CM - CN = 0$$

$$MW - \frac{\sqrt{2} \cdot \sqrt{2 \cdot AR - AR^2}}{2 \cdot \left(\sqrt{2 - AR} + \frac{\sqrt{2}}{2} \right)} = 0 \qquad MW - OV = 0$$

$m\angle STU = 33.55681°$

$m\angle STV = 100.67042°$

$\frac{m\angle STV}{m\angle STU} = 3.00000$

From 3 to 2.41421

$$AR - \frac{(BF + 1) \cdot (3 \cdot BF - 1)}{2 \cdot BF^2} = 0$$

$$G := \frac{1}{BF} \qquad AR - \frac{(G + 1) \cdot (3 - G)}{2} = 0$$

$N = 2.66668$

$W = 3.66668$

$(N + 1) = 3.66668$

$W - (N + 1) = 0.00000$

$X = 0.33332$

$(3 - N) = 0.33332$

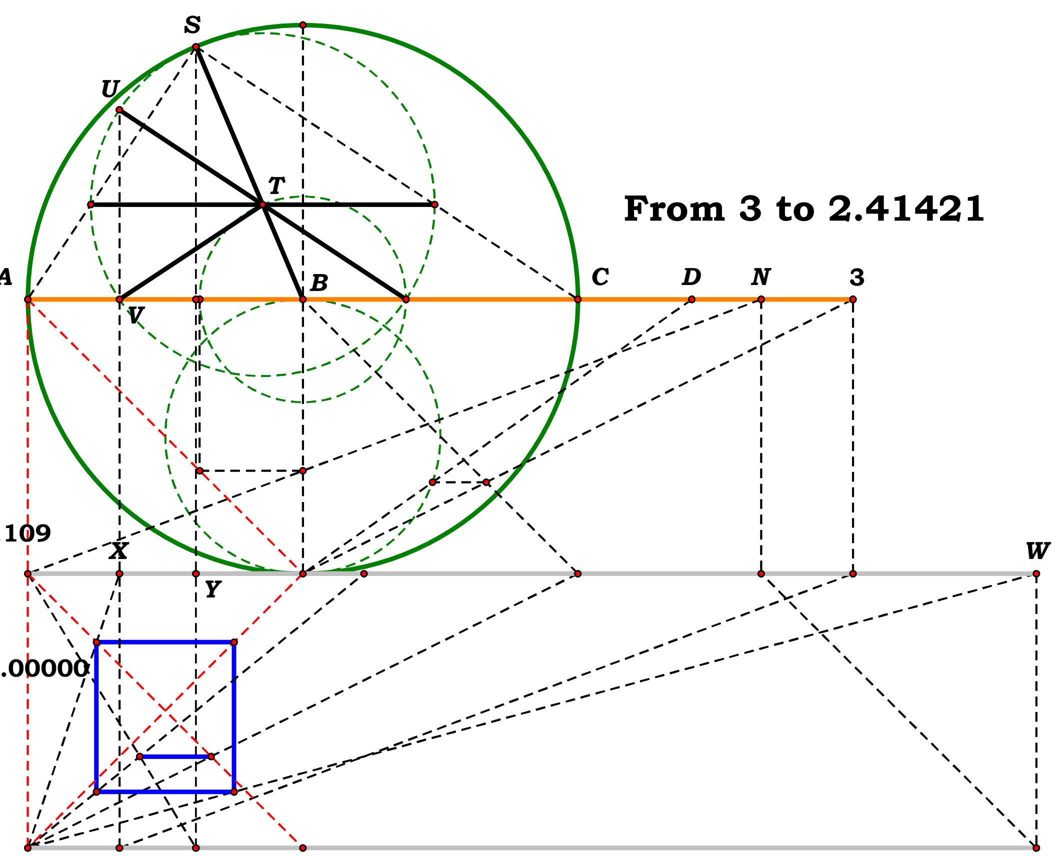
$X - (3 - N) = 0.00000$

$Y = 0.61109$

$\frac{(N + 1) \cdot (3 - N)}{2} = 0.61109$

$Y - \frac{(N + 1) \cdot (3 - N)}{2} = 0.00000$

$\frac{1}{N} = 0.37500$



042897 On Trisection

Descriptions.

$N := 1.69426 \qquad AB := 1 \qquad AC := \dfrac{1}{N}$

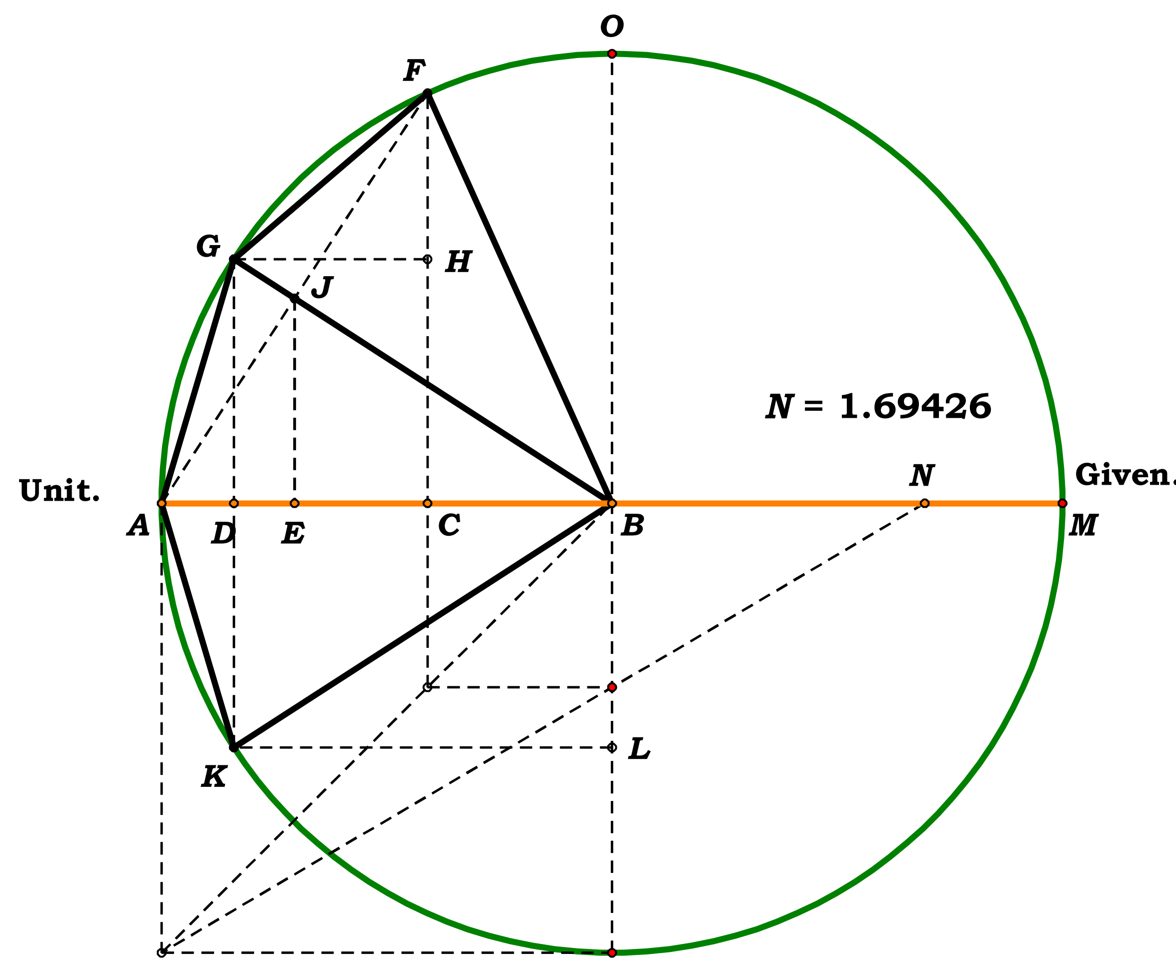
$AM := 2 \cdot AB \qquad CF := \sqrt{AC \cdot (AM - AC)}$

$AF := \sqrt{AC^2 + CF^2} \qquad AJ := \dfrac{AF}{2} \qquad AE := \dfrac{AC}{2}$

$EJ := \dfrac{CF}{2} \qquad BE := AB - AE \qquad BJ := \sqrt{BE^2 + EJ^2}$

$GJ := AB - BJ \qquad BD := \dfrac{BE}{BJ} \qquad DG := \dfrac{EJ \cdot BD}{BE}$

$FH := CF - DG$



Definitions.

$AB - 1 = 0 \qquad AC - \dfrac{1}{N} = 0 \qquad AM - 2 \cdot AB = 0 \qquad CF - \dfrac{\sqrt{2 \cdot N - 1}}{N} = 0 \qquad AF - \dfrac{\sqrt{2}}{\sqrt{N}} = 0$

$AJ - \dfrac{\sqrt{2}}{2 \cdot \sqrt{N}} = 0 \qquad AE - \dfrac{1}{2 \cdot N} = 0 \qquad EJ - \dfrac{\sqrt{2 \cdot N - 1}}{2 \cdot N} = 0 \qquad BE - \dfrac{2 \cdot N - 1}{2 \cdot N} = 0$

$BJ - \dfrac{\sqrt{4 \cdot N - 2}}{2 \cdot \sqrt{N}} = 0 \qquad GJ - \dfrac{2 \cdot \sqrt{N} - \sqrt{2} \cdot \sqrt{2 \cdot N - 1}}{2 \cdot \sqrt{N}} = 0 \qquad BD - \dfrac{\sqrt{4 \cdot N - 2}}{2 \cdot \sqrt{N}} = 0 \qquad DG - \dfrac{\sqrt{2}}{2 \cdot \sqrt{N}} = 0$

$FH - \dfrac{2 \cdot \sqrt{2 \cdot N - 1} - \sqrt{2} \cdot \sqrt{N}}{2 \cdot N} = 0$

Man in the Middle

Descriptions.

$AN := 3.53349 \quad AC := 1 \quad AB := \frac{1}{\sqrt{AN}} \quad EF := AC - AB$

$CF := \sqrt{1 - EF^2} \quad AF := AC - CF \quad CG := CF \quad AM := 2 \cdot AC$

$GH := EF \cdot CG \quad JK := 2 \cdot GH \quad KM := \frac{CF \cdot JK}{EF} \quad AK := AM - KM$

Definitions.

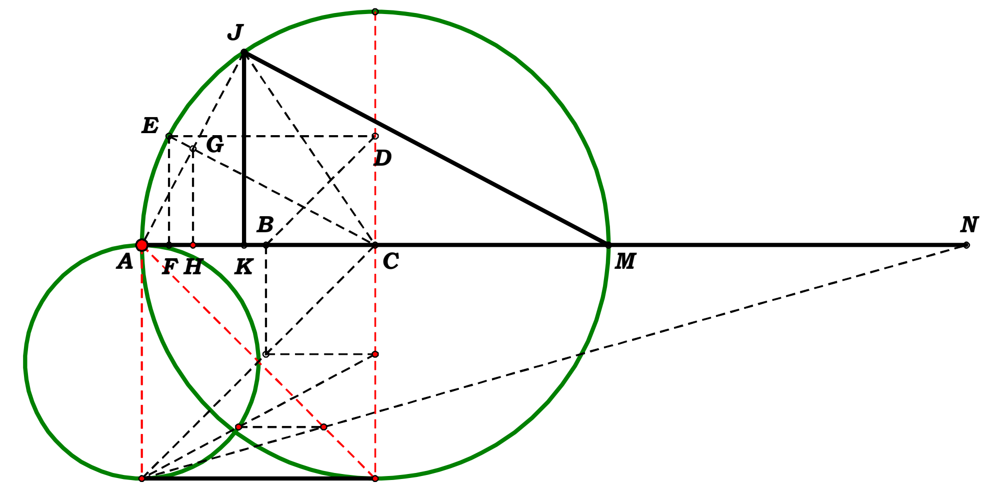
$AN - AN = 0 \quad AC - 1 = 0 \quad AB - \frac{1}{\sqrt{AN}} = 0$

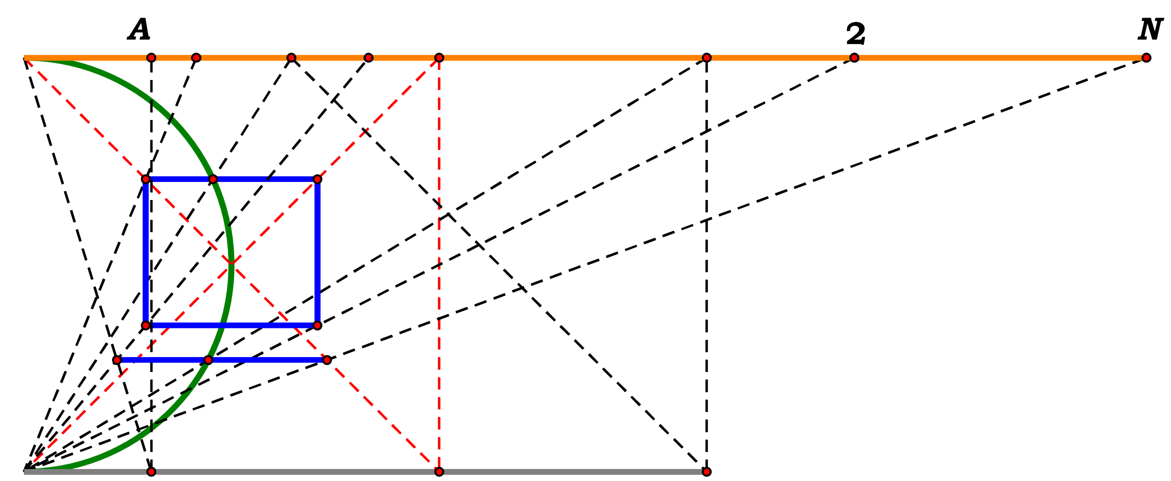
$EF - \frac{\sqrt{AN} - 1}{\sqrt{AN}} = 0 \quad CF - \frac{\sqrt{2 \cdot \sqrt{AN} - 1}}{\sqrt{AN}} = 0$

$AF - \frac{\sqrt{AN} - \sqrt{2 \cdot \sqrt{AN} - 1}}{\sqrt{AN}} = 0 \quad CG - CF = 0 \quad AM - 2 = 0 \quad GH - \frac{\sqrt{2 \cdot \sqrt{AN} - 1} \cdot \left(\sqrt{AN} - 1\right)}{AN} = 0$

$JK - \frac{2 \cdot \sqrt{2 \cdot \sqrt{AN} - 1} \cdot \left(\sqrt{AN} - 1\right)}{AN} = 0 \quad KM - \frac{2 \cdot \left(2 \cdot \sqrt{AN} - 1\right)}{AN} = 0 \quad AK - \frac{2 \cdot \left(\sqrt{AN} - 1\right)^2}{AN} = 0$

$N = 3.53349$
$A = 0.00000$
$B = 0.53198$
$C = 1.00000$
$\frac{1}{\sqrt{N}} = 0.53198$
$D = 0.46802$
$G = 0.41360$
$K = 0.43808$





$N = 2.70326$

$\frac{2 \cdot \left(\sqrt{N} - 1\right)^2}{N} = 0.30699$

$A = 0.30699$

$A - \frac{2 \cdot \left(\sqrt{N} - 1\right)^2}{N} = 0.00000$

On Trisection 042897

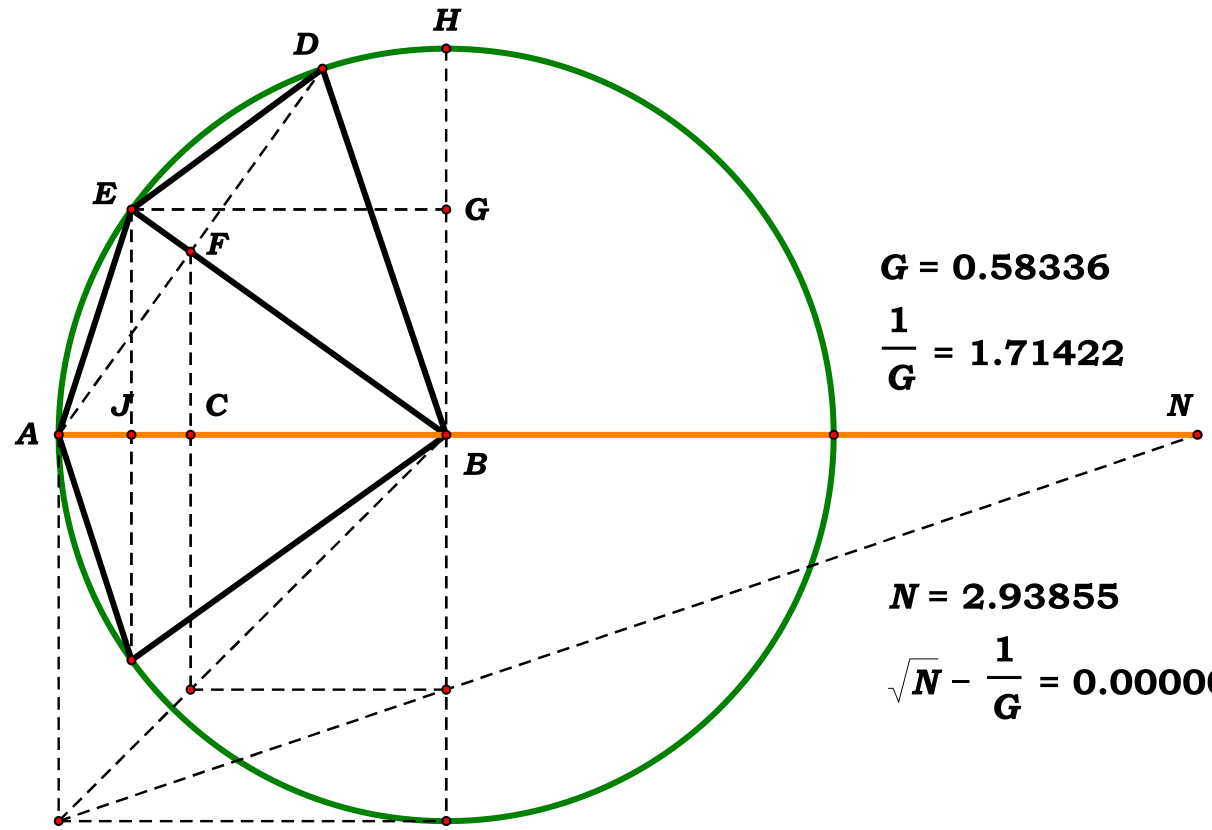
Descriptions.

$BG := .58336 \quad AB := 1 \quad GH := AB - BG$

$EG := \sqrt{GH \cdot (2 \cdot AB - GH)} \quad AJ := AB - EG$

$BF := AB - AJ \quad BJ := AB - AJ \quad BC := BJ \cdot BF$

$AC := AB - BC \quad AN := \dfrac{1}{AC} \quad \dfrac{1}{BG} - \sqrt{AN} = 0$



$G = 0.58336$

$\dfrac{1}{G} = 1.71422$

$N = 2.93855$

$\sqrt{N} - \dfrac{1}{G} = 0.00000$

Definitions.

$BG - BG = 0 \quad AB - 1 = 0 \quad GH - (1 - BG) = 0 \quad EG - \sqrt{\left(1 - BG^2\right)} = 0$

$AJ - \left(1 - \sqrt{1 - BG^2}\right) \quad BF - \left(\sqrt{1 - BG^2}\right) \quad BJ - \sqrt{1 - BG^2} = 0 \quad BC - \left(1 - BG^2\right)$

$AC - BG^2 \quad AN - \dfrac{1}{BG^2} = 0 \quad \dfrac{1}{BG} - \sqrt{AN} = 0$

Trisection and Cube Roots

Descriptions.

$AB := 1 \quad AN := 7 \quad BN := AN - AB \quad AC := AN^{\frac{1}{3}} \quad AE := AN^{\frac{2}{3}}$

$CE := AE - AC \quad CD := \frac{CE}{2} \quad AD := AC + CD \quad AG := CD$

$DG := \sqrt{AD^2 + AG^2} \quad CM := DG \quad MO := AG \quad JM := \frac{MO^2}{DG}$

$JO := \sqrt{MO^2 - JM^2} \quad RS := AG + 2 \cdot JO \quad AS := DG - 2 \cdot JM$

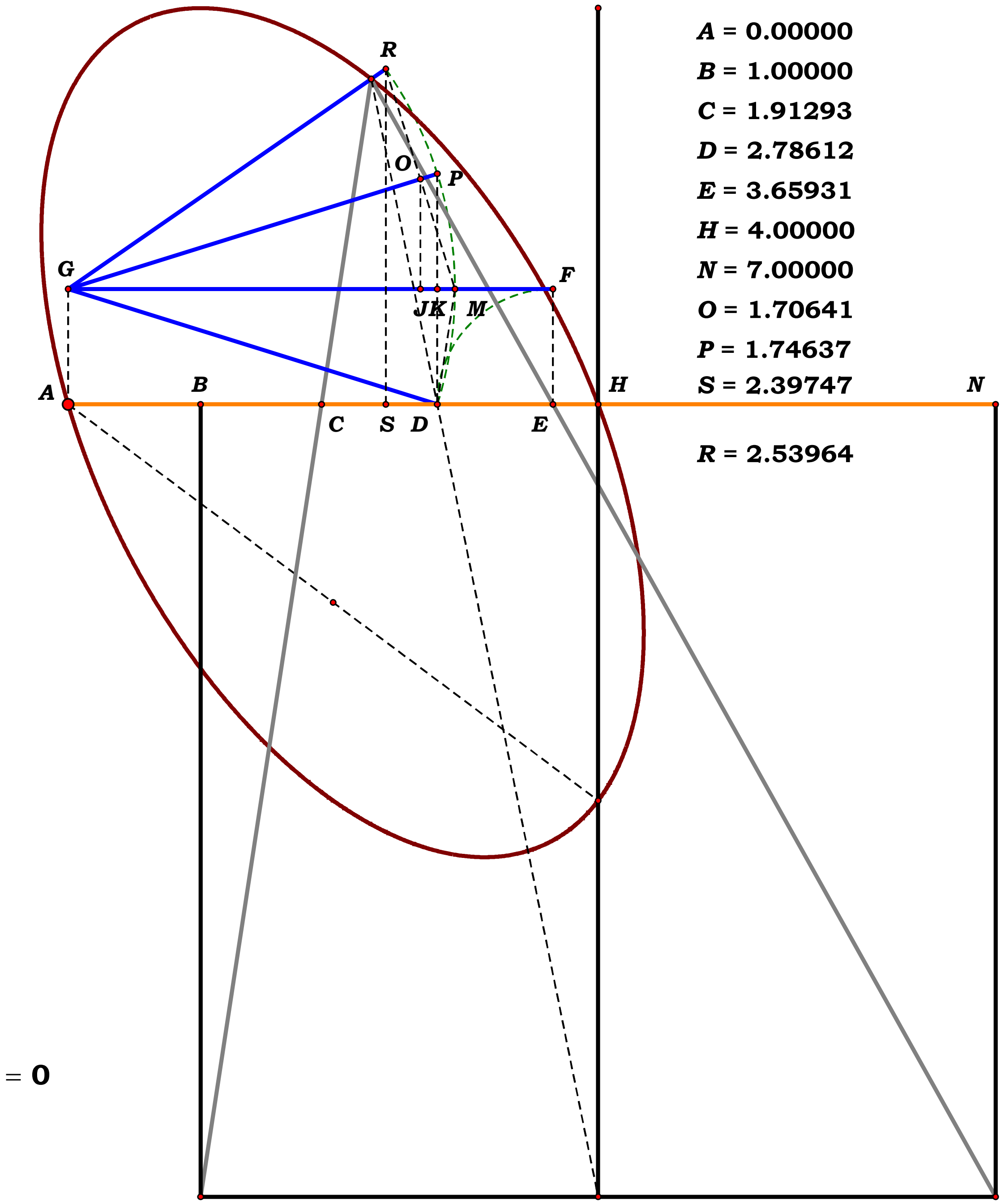
Definitions.

$AB - 1 = 0 \quad AN - AN = 0 \quad BN - (AN - 1) = 0 \quad AC - AN^{\frac{1}{3}} = 0$

$AE - AN^{\frac{2}{3}} = 0 \quad CE - \left(AN^{\frac{2}{3}} - AN^{\frac{1}{3}}\right) = 0 \quad CD - \frac{AN^{\frac{2}{3}} - AN^{\frac{1}{3}}}{2} = 0$

$AD - \frac{AN^{\frac{1}{3}} \cdot \left(AN^{\frac{1}{3}} + 1\right)}{2} = 0 \quad AG - CD = 0 \quad DG - \frac{\sqrt{AN^{\frac{2}{3}} \cdot \left(AN^{\frac{2}{3}} + 1\right)}}{\sqrt{2}} = 0$

$CM - DG = 0 \quad MO - AG = 0$



A = 0.00000
B = 1.00000
C = 1.91293
D = 2.78612
E = 3.65931
H = 4.00000
N = 7.00000
O = 1.70641
P = 1.74637
S = 2.39747

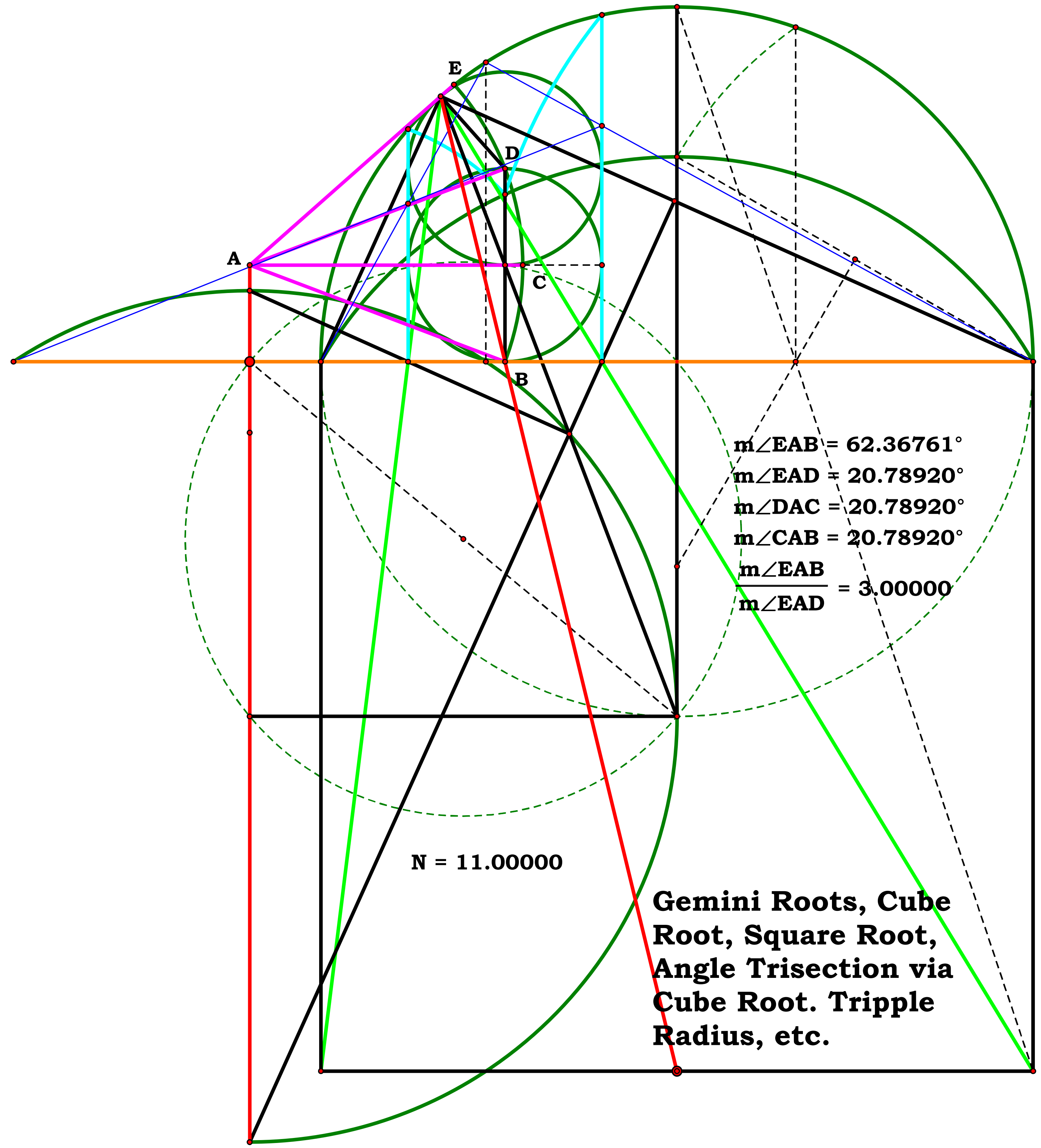
R = 2.53964

$JM - \frac{\sqrt{2} \cdot AN^{\frac{2}{3}} \cdot \left(AN^{\frac{1}{3}} - 1\right)^2}{4 \cdot \sqrt{AN^{\frac{2}{3}} + AN^{\frac{4}{3}}}} = 0 \quad JO - \frac{AN^{\frac{1}{3}} \cdot \left(AN^{\frac{2}{3}} - 1\right)}{2 \cdot \sqrt{2 \cdot \left(AN^{\frac{2}{3}} + 1\right)}} = 0$

$RS - \left[\frac{AN^{\frac{1}{3}} \cdot \left(AN^{\frac{1}{3}} - 1\right) \cdot \left(\sqrt{2} \cdot AN^{\frac{1}{3}} + \sqrt{AN^{\frac{2}{3}} + 1} + \sqrt{2}\right)}{2 \cdot \sqrt{AN^{\frac{2}{3}} + 1}}\right] = 0 \quad AS - \frac{\sqrt{2} \cdot AN}{\sqrt{AN^{\frac{2}{3}} + AN^{\frac{4}{3}}}} = 0$

Trisection and Cube Roots

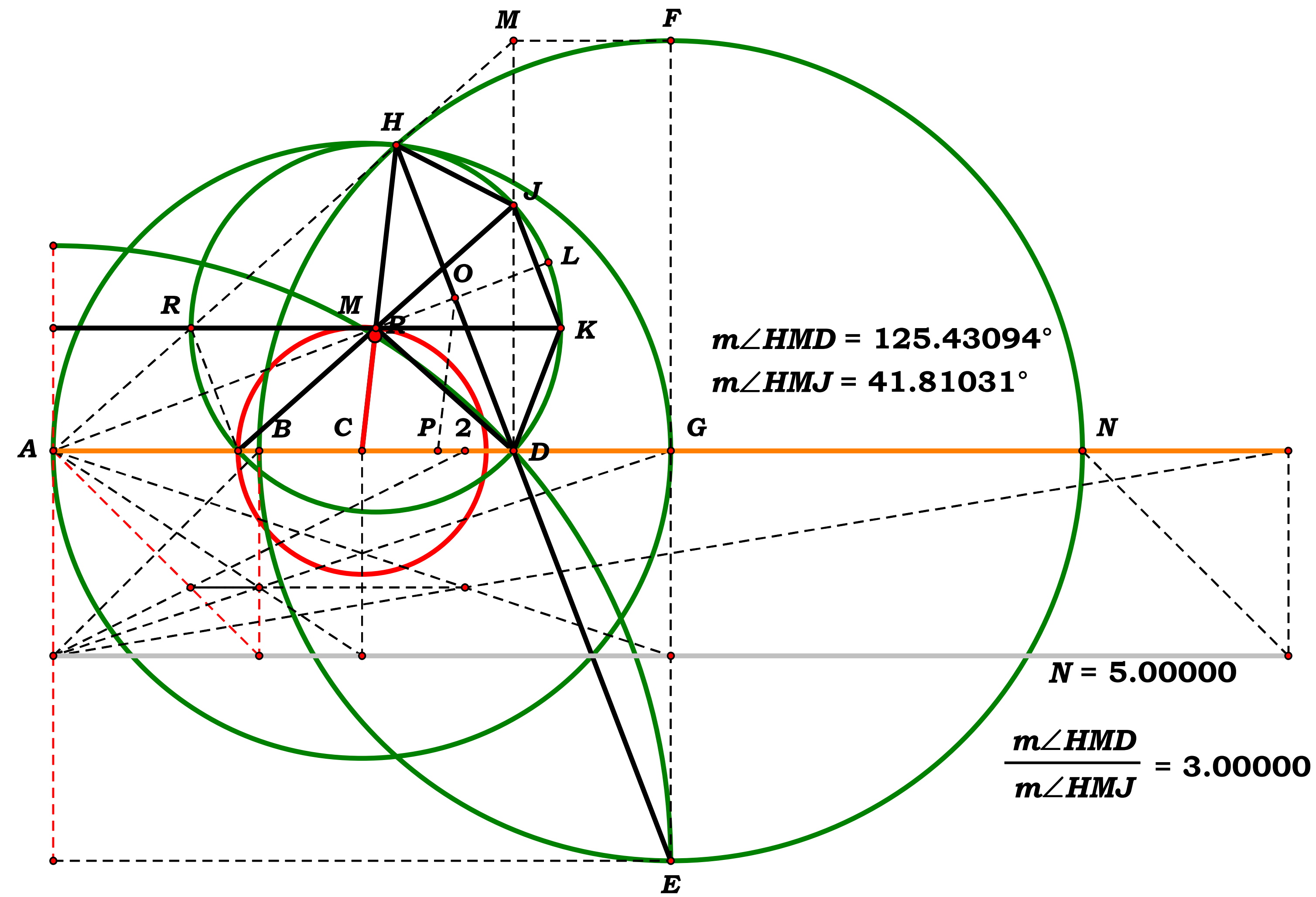
One has to be careful with drawing this for point E is not on the circle, so in drawingit, one might want to hid the circle it is almost on top of.



Paper Roots

Unit.

Given.



Descriptions.

$AN := 5 \quad AD := \sqrt{AN} \quad AB := 1 \quad BN := AN - AB$

$BG := \dfrac{BN}{2} \quad AG := AB + BG \quad AC := \dfrac{AG}{2} \quad CH := AC$

$DG := AG - AD \quad DE := \sqrt{DG^2 + BG^2} \quad EH := \dfrac{BG \cdot BN}{DE}$

$DH := EH - DE \quad DO := \dfrac{DH}{2} \quad OP := \dfrac{CH \cdot DO}{DH}$

$CD := AD - AC \quad CP := \dfrac{CD}{2} \quad AP := AC + CP \quad CM := \dfrac{OP \cdot AC}{AP}$

$AH := AD \quad AR := \dfrac{AH \cdot CM}{CH} \quad HM := CH - CM \quad HM - AR = 0$

Definitions.

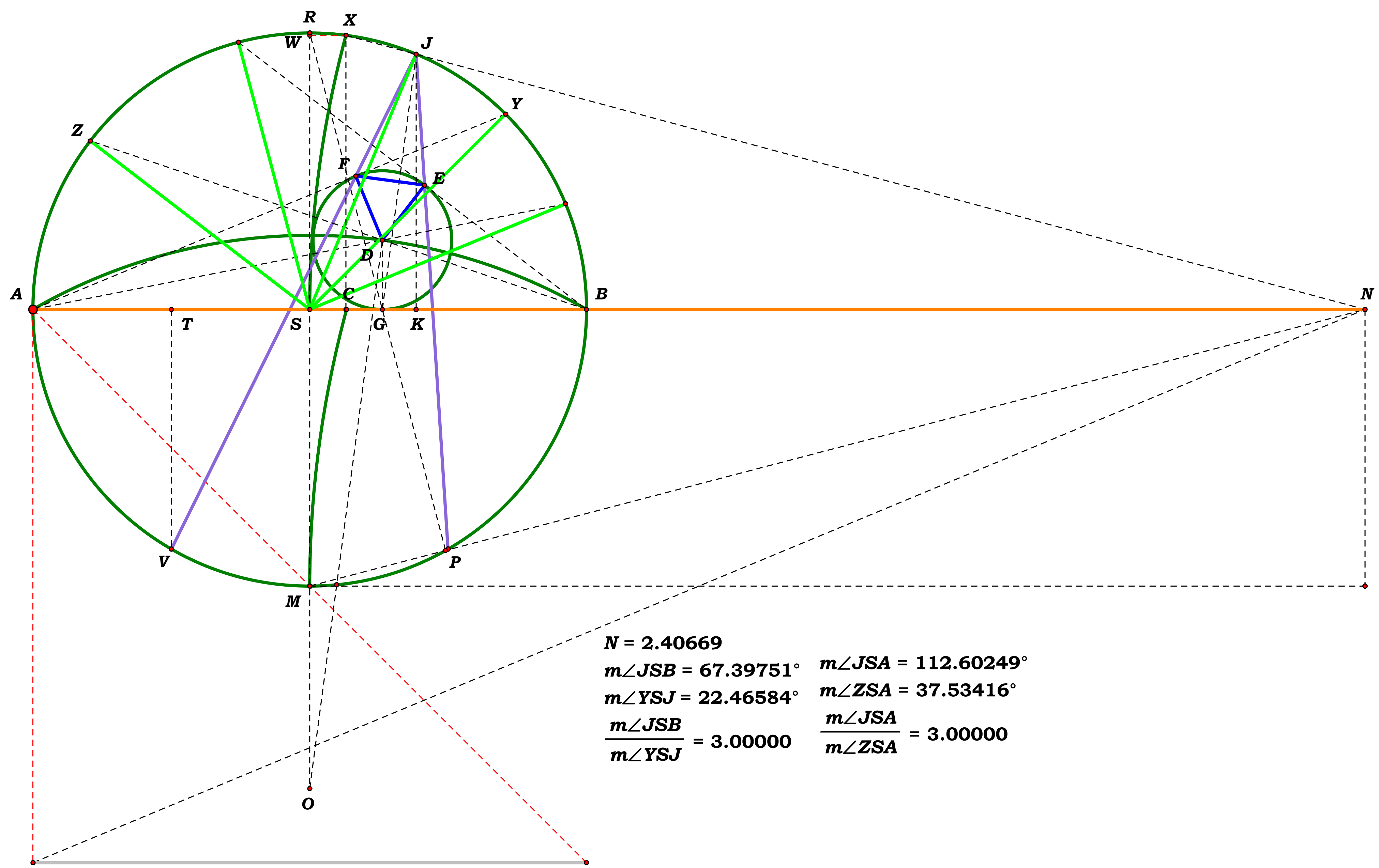
$AN - AN = 0 \quad AD - \sqrt{AN} = 0 \quad AB - 1 = 0 \quad BN - (AN - 1) = 0 \quad BG - \dfrac{AN - 1}{2} = 0 \quad AG - \dfrac{AN + 1}{2} = 0 \quad AC - \dfrac{AN + 1}{4} = 0 \quad CH - AC = 0$

$DG - \dfrac{\left(\sqrt{AN} - 1\right)^2}{2} = 0 \quad DE - \dfrac{\left(\sqrt{AN} - 1\right) \cdot \sqrt{(AN + 1)}}{\sqrt{2}} = 0 \quad EH - \dfrac{\sqrt{2} \cdot \left(\sqrt{AN} - 1\right) \cdot \left(\sqrt{AN} + 1\right)^2}{2 \cdot \sqrt{AN + 1}} = 0 \quad DH - \dfrac{\sqrt{2} \cdot \sqrt{AN} \cdot \left(\sqrt{AN} - 1\right)}{\sqrt{AN + 1}} = 0$

$DO - \dfrac{\sqrt{2} \cdot \sqrt{AN} \cdot \left(\sqrt{AN} - 1\right)}{2 \cdot \sqrt{AN + 1}} = 0 \quad OP - \dfrac{AN + 1}{8} = 0 \quad CD - \dfrac{\left(4 \cdot \sqrt{AN} - AN - 1\right)}{4} = 0 \quad CP - \dfrac{\left(4 \cdot \sqrt{AN} - AN - 1\right)}{8} = 0 \quad AP - \dfrac{AN + 4 \cdot \sqrt{AN} + 1}{8} = 0$

$CM - \dfrac{(AN + 1)^2}{4 \cdot \left(AN + 4 \cdot \sqrt{AN} + 1\right)} = 0 \quad AH - AD = 0 \quad AR - \dfrac{\sqrt{AN} \cdot (AN + 1)}{AN + 4 \cdot \sqrt{AN} + 1} = 0 \quad HM - \dfrac{\sqrt{AN} \cdot (AN + 1)}{AN + 4 \cdot \sqrt{AN} + 1} = 0 \quad HM - AR = 0$

Angle Trisection



$N = 2.40669$

$m\angle JSB = 67.39751°$

$m\angle YSJ = 22.46584°$

$\dfrac{m\angle JSB}{m\angle YSJ} = 3.00000$

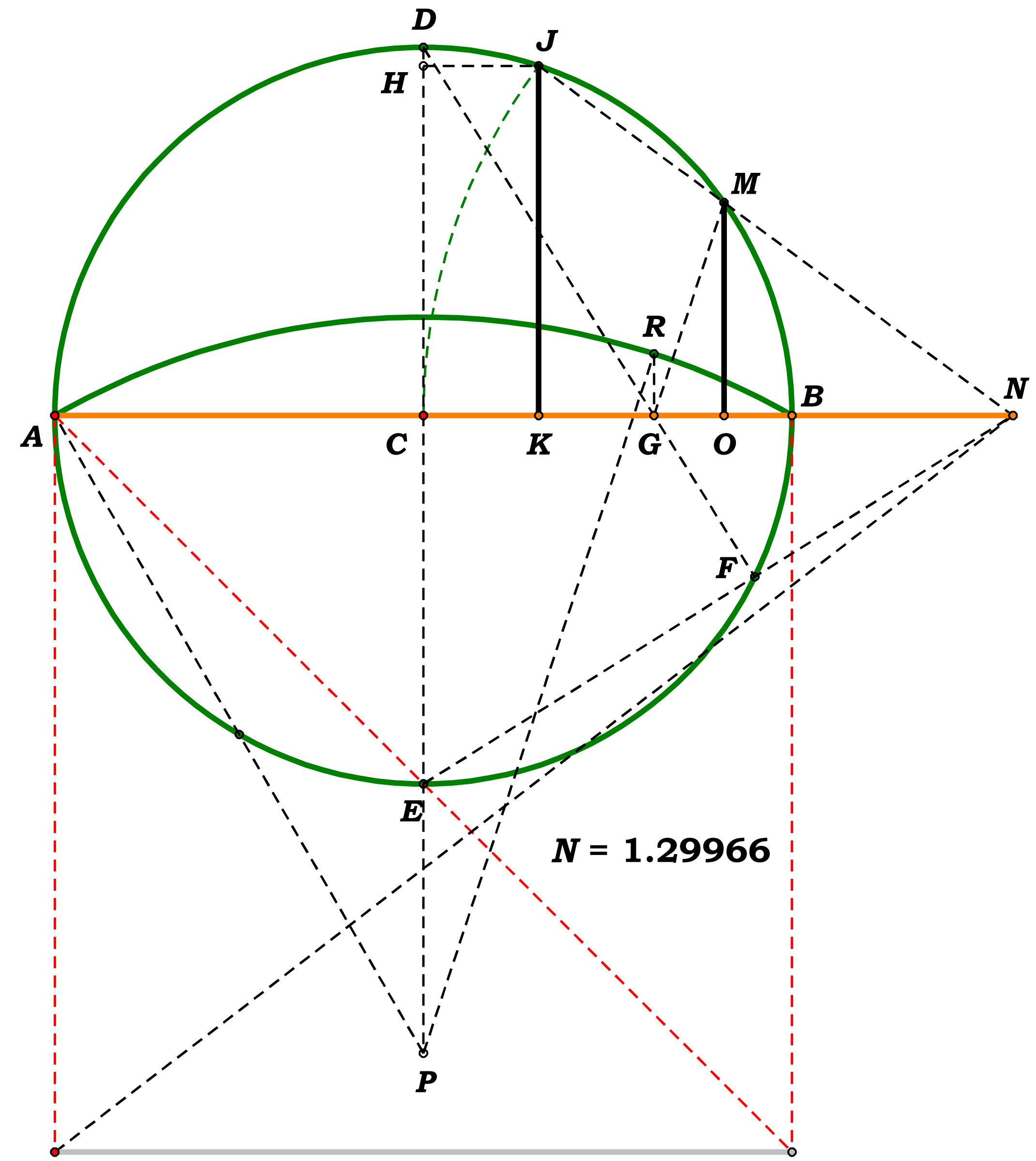
$m\angle JSA = 112.60249°$

$m\angle ZSA = 37.53416°$

$\dfrac{m\angle JSA}{m\angle ZSA} = 3.00000$

Angle Trisection

Unit.
Given.



Descriptions.

$$AB := 1 \quad AN := 1.29966 \quad AC := \frac{AB}{2} \quad CN := AN - AC$$

$$CG := \frac{AC^2}{CN} \quad S_1 := CN \quad S_2 := S_1 \quad S_3 := AC$$

$$JK := \frac{\sqrt{(S_1 + S_2 - S_3)\cdot(S_1 - S_2 + S_3)\cdot(S_2 - S_1 + S_3)\cdot(S_1 + S_2 + S_3)}}{2\cdot S_1}$$

$$KN := \sqrt{CN^2 - JK^2} \quad CK := CN - KN \quad MN := CN - 2\cdot CK$$

$$NO := \frac{KN\cdot MN}{CN}$$

Definitions.

$$AB - AB = 0 \quad AN - AN = 0 \quad AC - \frac{1}{2} = 0 \quad CN - \frac{2\cdot AN - 1}{2} = 0$$

$$CG - \frac{1}{2\cdot(2\cdot AN - 1)} = 0 \quad S_1 - CN = 0 \quad S_2 - S_1 = 0 \quad S_3 - AC = 0$$

$$JK - \frac{\sqrt{(4\cdot AN - 1)\cdot(4\cdot AN - 3)}}{4\cdot(2\cdot AN - 1)} = 0 \quad KN - \frac{\left(8\cdot AN^2 - 8\cdot AN + 1\right)}{4\cdot(2\cdot AN - 1)} = 0 \quad CK - \frac{1}{4\cdot(2\cdot AN - 1)} = 0 \quad MN - \frac{2\cdot AN\cdot(AN - 1)}{2\cdot AN - 1} = 0$$

$$NO - \frac{AN\cdot(AN - 1)\cdot\left(8\cdot AN^2 - 8\cdot AN + 1\right)}{(2\cdot AN - 1)^3} = 0$$

Which Mathcad says is solvable but perhaps too much for my pc. Or, contrary to popular belief, any angle can be trisected. So, not only can you draw it, you can, if you desire, do it with algebra because you can even draw the equations with geometry.

Three Quarter Boot

Descriptions.

$AB := 1 \quad AN := 3 \quad AC := \dfrac{1}{AN} \quad BC := AB - AC \quad CD := \dfrac{BC}{2}$

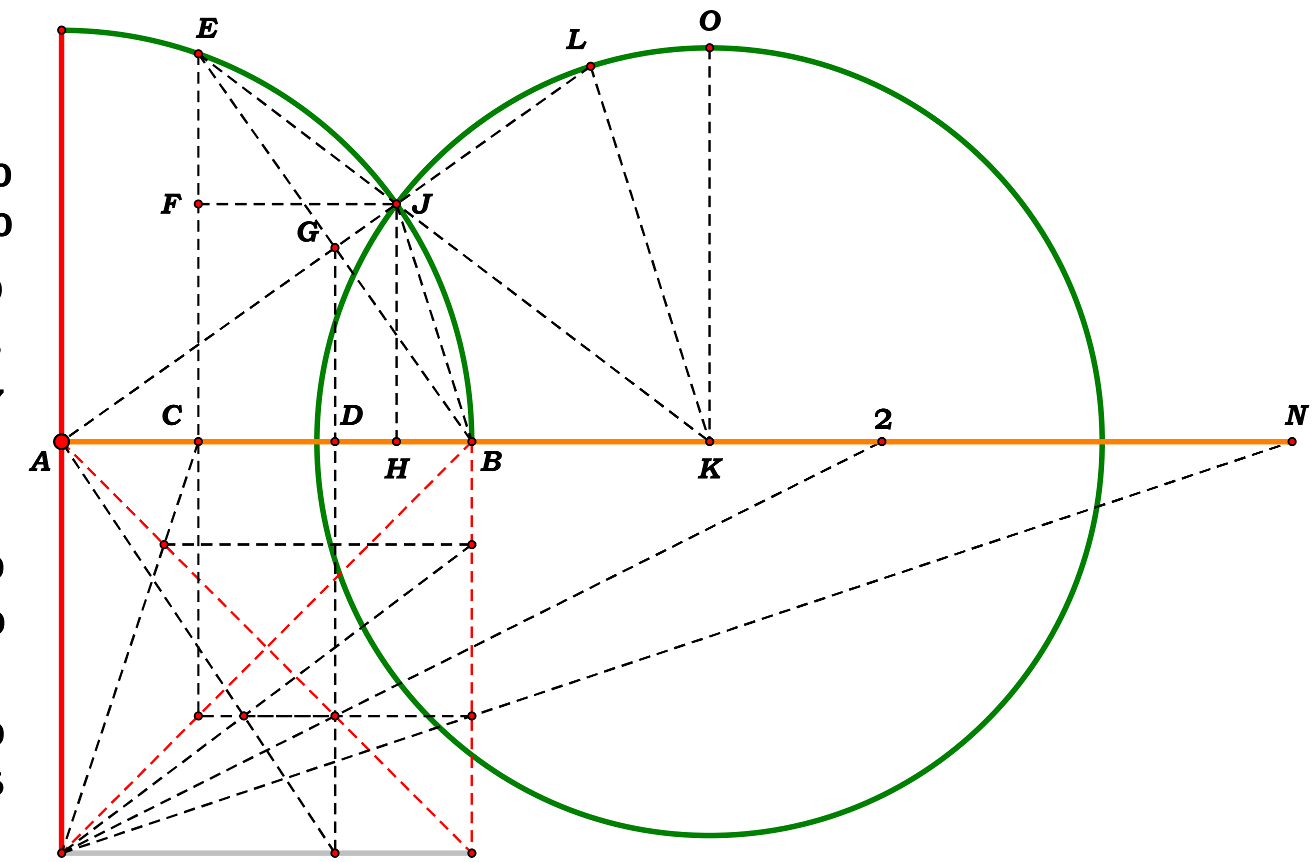
$CE := \sqrt{BC\cdot(2\cdot AB - BC)} \quad DG := \dfrac{CE}{2} \quad AD := AC + CD$

$AG := \sqrt{AD^2 + DG^2} \quad AH := \dfrac{AD}{AG} \quad HJ := \dfrac{DG}{AG}$

$EF := CE - HJ \quad FJ := AH - AC \quad CK := \dfrac{FJ\cdot CE}{EF} \quad AK := CK + AC$

$EJ := \sqrt{EF^2 + FJ^2} \quad HK := AK - AH \quad JK := \dfrac{EJ\cdot HJ}{EF} \quad JK = 0.957058$

$A = 0.00000$
$N = 3.00000$
$B = 1.00000$
$C = 0.33333$
$D = 0.66667$
$E = 0.94281$
$F = 0.57735$
$G = 0.47140$
$H = 0.81650$
$J = 0.57735$
$K = 1.57980$
$O = 0.95706$



Definitions.

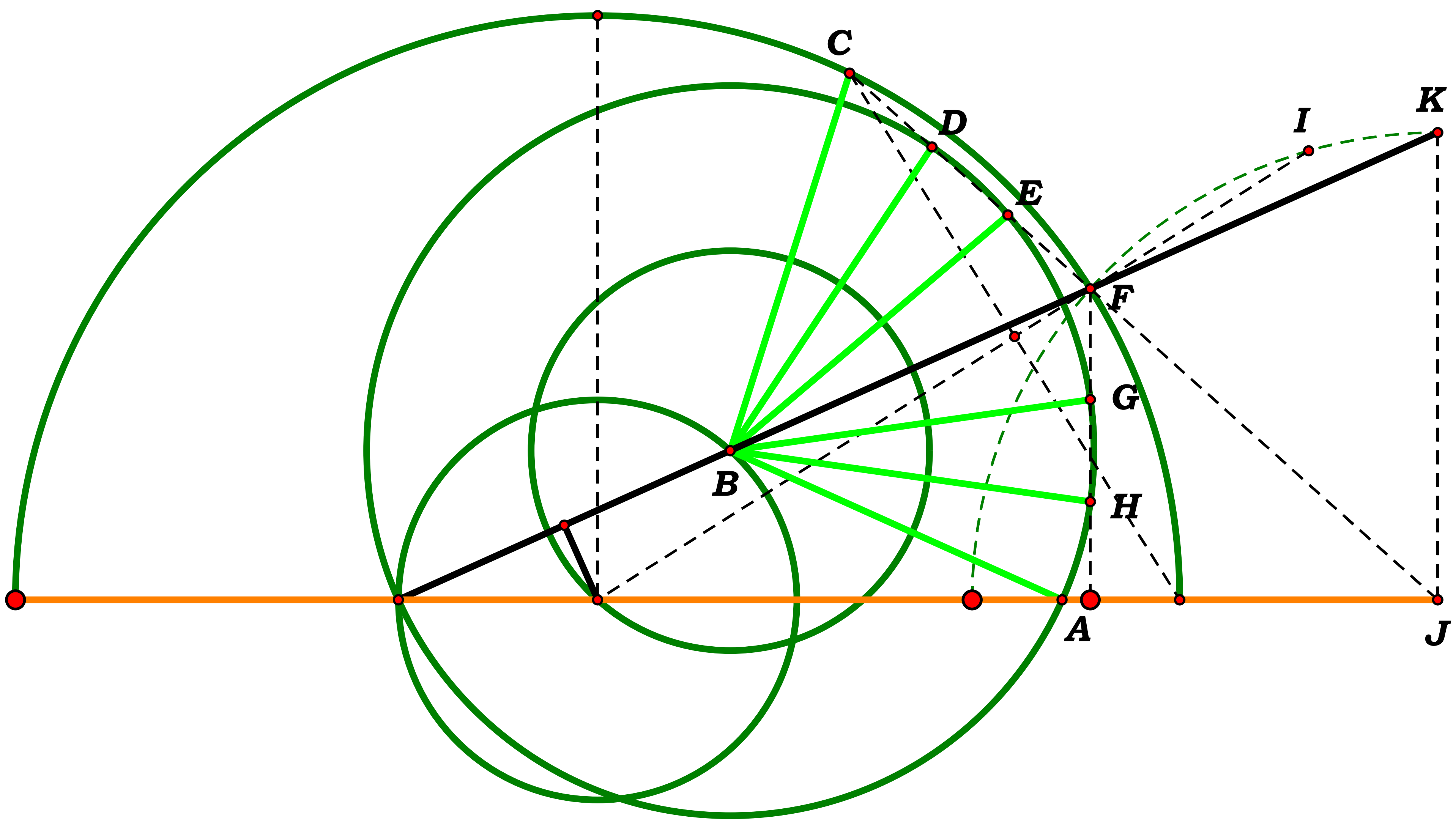
$AB - AB = 0 \quad AN - AN = 0 \quad AC - \dfrac{1}{AN} = 0 \quad BC - \dfrac{AN-1}{AN} = 0 \quad CD - \dfrac{AN-1}{2\cdot AN} = 0 \quad CE - \dfrac{\sqrt{(AN-1)\cdot(AN+1)}}{AN} = 0 \quad DG - \dfrac{\sqrt{(AN-1)\cdot(AN+1)}}{2\cdot AN} = 0 \quad AD - \dfrac{AN+1}{2\cdot AN} = 0$

$AG - \dfrac{\sqrt{AN+1}}{\sqrt{2\cdot AN}} = 0 \quad AH - \dfrac{\sqrt{2}\cdot\sqrt{AN+1}}{2\cdot\sqrt{AN}} = 0 \quad HJ - \dfrac{\sqrt{2}\cdot\sqrt{AN^2-1}}{2\cdot\sqrt{AN}\cdot\sqrt{AN+1}} = 0 \quad EF - \dfrac{\sqrt{AN^2-1}\cdot\left(2\cdot\sqrt{AN+1}-\sqrt{2}\cdot\sqrt{AN}\right)}{2\cdot AN\cdot\sqrt{AN+1}} = 0 \quad FJ - \dfrac{\sqrt{2}\cdot\sqrt{AN}\cdot\sqrt{AN+1}-2}{2\cdot AN} = 0$

$CK - \dfrac{\sqrt{AN+1}\cdot\sqrt{(AN-1)\cdot(AN+1)}\cdot\left(\sqrt{2}\cdot\sqrt{AN}\cdot\sqrt{AN+1}-2\right)}{AN\cdot\left(2\cdot\sqrt{AN+1}-\sqrt{2}\cdot\sqrt{AN}\right)\cdot\sqrt{AN^2-1}} = 0 \quad AK - \dfrac{\sqrt{2\cdot AN}}{2\cdot\sqrt{AN+1}-\sqrt{2\cdot AN}} = 0 \quad EJ - \dfrac{\sqrt{2\cdot AN^{\frac{3}{2}} + 2\cdot AN^{\frac{5}{2}} + \sqrt{2}\cdot\sqrt{AN+1} - \sqrt{2}\cdot(AN+1)^{\frac{3}{2}} - \sqrt{2}\cdot AN^2\cdot\sqrt{AN+1}}}{\sqrt{AN^{\frac{3}{2}}\cdot(AN+1)}} = 0$

$HK - \dfrac{\sqrt{AN}\cdot\sqrt{AN+1}-\sqrt{2}}{\sqrt{AN}\cdot\left(2\cdot\sqrt{AN+1}-\sqrt{2}\cdot\sqrt{AN}\right)} = 0 \quad JK - \dfrac{\sqrt{2}\cdot\sqrt{AN}\cdot\sqrt{\sqrt{2\cdot AN+2} + 2\cdot AN^{\frac{3}{2}} + 2\cdot AN^{\frac{5}{2}} - AN^2\cdot\sqrt{2\cdot AN+2} - \sqrt{2}\cdot(AN+1)^{\frac{3}{2}}}}{\left(2\cdot\sqrt{AN+1}-\sqrt{2\cdot AN}\right)\cdot\sqrt{AN^{\frac{3}{2}}\cdot(AN+1)}} = 0$

Foot Note to Three Quarter Boot



$m\angle ABC = 96.52156°$

$m\angle CBD = 16.08693°$

$m\angle DBE = 16.08693°$

$m\angle EBF = 16.08693°$

$m\angle FBG = 16.08693°$

$m\angle GBH = 16.08693°$

$m\angle HBA = 16.08693°$

$m\angle IJK = 16.08693°$

$$\frac{m\angle ABC}{m\angle CBD} = 6.00000$$

042401

Descriptions.

$$AB := 1 \quad AN := 4 \quad BN := AN - AB \quad BF := \frac{BN}{2}$$

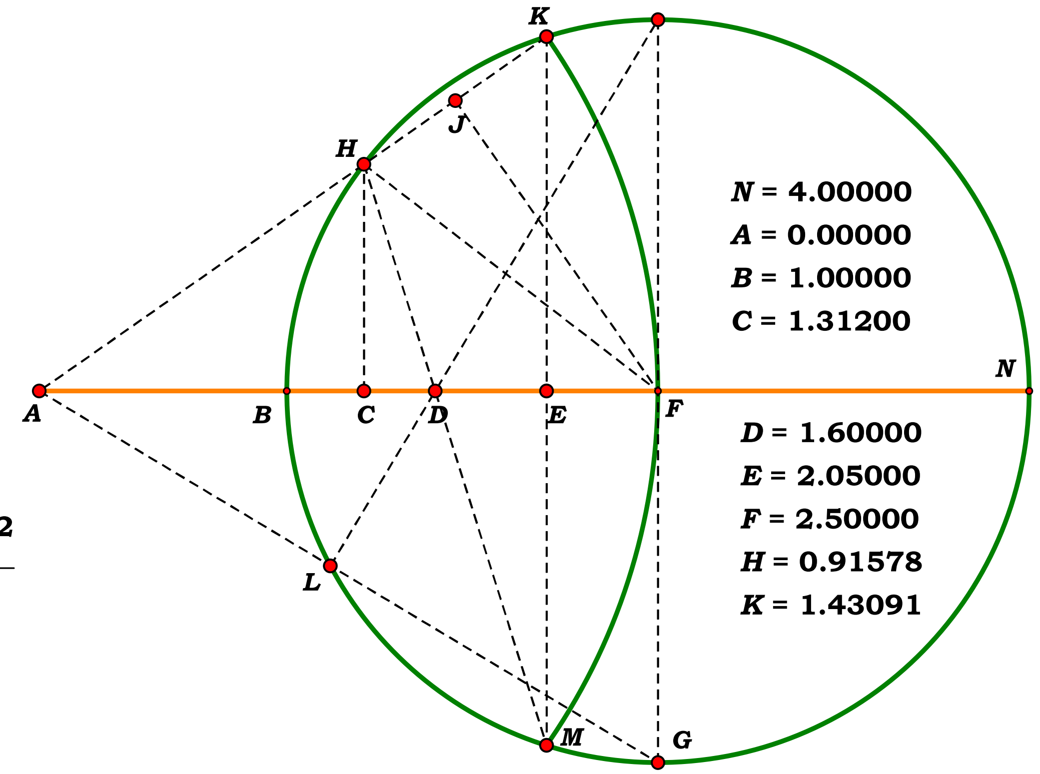
$$AF := AB + BF \quad AK := AF \quad FK := BF \quad AE := \frac{2 \cdot AK^2 - FK^2}{2 \cdot AF}$$

See Pythagoras Revisted in the Delian Quest for AE.

$$AJ := AE \quad JK := AK - AJ \quad HJ := JK \quad AH := AK - 2 \cdot JK$$

$$AC := \frac{AE \cdot AH}{AK} \quad CE := AE - AC \quad BE := AE - AB \quad EN := BN - BE$$

$$EK := \sqrt{BE \cdot EN} \quad CH := \frac{EK \cdot AH}{AK} \quad DE := \frac{CE \cdot EK}{EK + CH} \quad DF := 2 \cdot DE \quad HM := \sqrt{CE^2 + (EK + CH)^2}$$



One should recognise the core of the figure for angle trisection. This is a simple write-up of relationships.

And, as the figure is wholly expressible algebraically, one can solve for more than one point by which the figure can be solved and reproduced for any angle one wishes to trisect, as I said before.

Definitions.

$$AB - AB = 0 \quad AN - AN = 0 \quad BN - (AN - 1) = 0 \quad BF - \frac{AN - 1}{2} = 0 \quad AF - \frac{AN + 1}{2} = 0 \quad AK - AF = 0 \quad FK - BF = 0 \quad AE - \frac{AN^2 + 6 \cdot AN + 1}{4 \cdot (AN + 1)} = 0$$

$$AJ - AE = 0 \quad JK - \frac{(AN - 1)^2}{4 \cdot (AN + 1)} = 0 \quad HJ - JK = 0 \quad AH - \frac{2 \cdot AN}{AN + 1} = 0 \quad AC - \frac{AN \cdot \left(AN^2 + 6 \cdot AN + 1\right)}{(AN + 1)^3} = 0 \quad CE - \frac{\left(AN^2 + 6 \cdot AN + 1\right) \cdot (AN - 1)^2}{4 \cdot (AN + 1)^3} = 0$$

$$BE - \frac{(AN + 3) \cdot (AN - 1)}{4 \cdot (AN + 1)} = 0 \quad EN - \frac{(3 \cdot AN + 1) \cdot (AN - 1)}{4 \cdot (AN + 1)} = 0 \quad EK - \frac{(AN - 1) \cdot \sqrt{(AN + 3) \cdot (3 \cdot AN + 1)}}{(4 \cdot AN + 4)} = 0 \quad CH - \frac{AN \cdot (AN - 1) \cdot \sqrt{(AN + 3) \cdot (3 \cdot AN + 1)}}{(AN + 1)^3} = 0$$

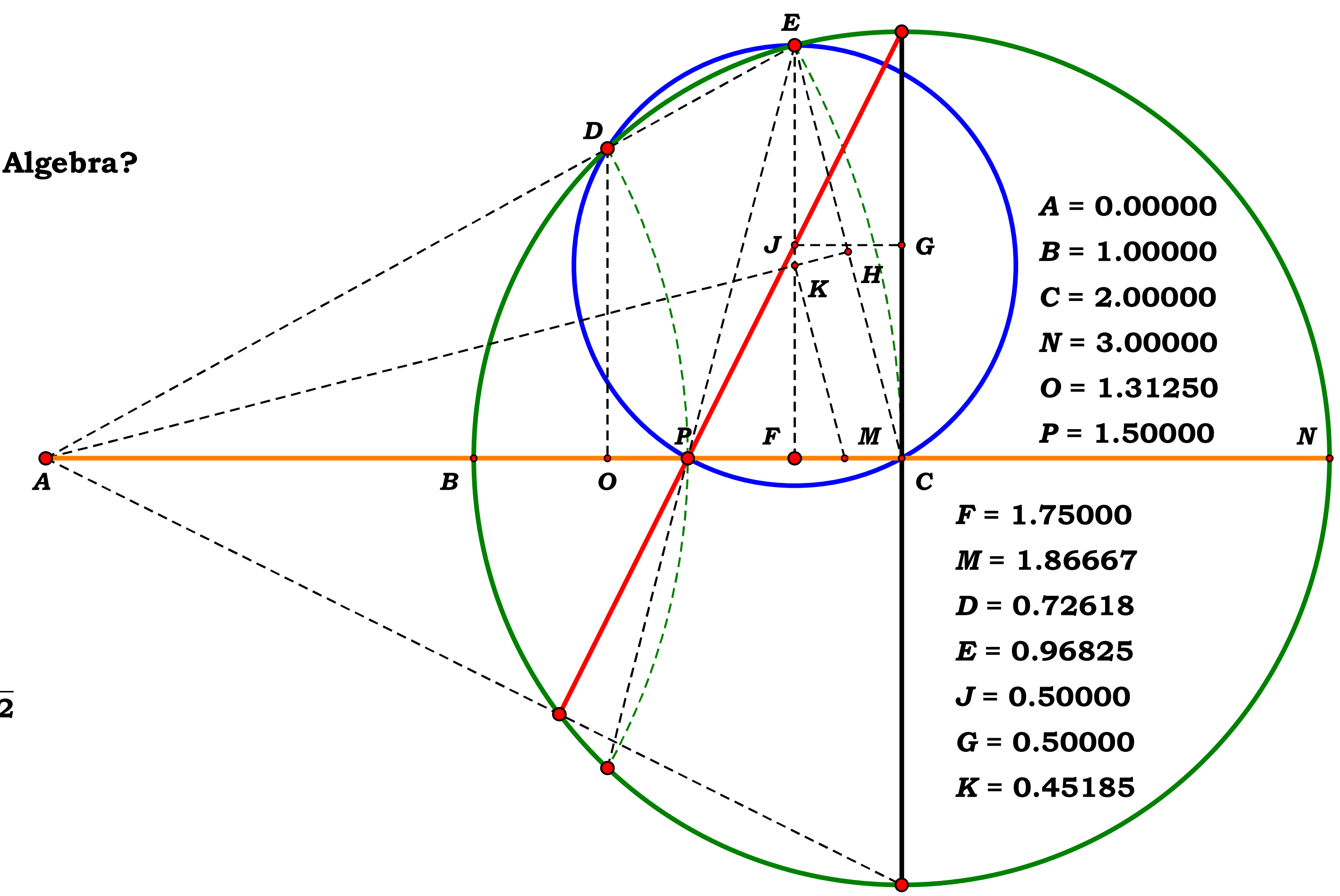
$$DE - \frac{(AN - 1)^2}{4 \cdot (AN + 1)} = 0 \quad DF - \frac{(AN - 1)^2}{2 \cdot (AN + 1)} = 0 \quad HM - \frac{(AN - 1) \cdot \left(AN^2 + 6 \cdot AN + 1\right)}{2 \cdot (AN + 1)^2} = 0$$

What is the Name 042501

Unit.

Given.

What is the name of EK in Algebra?



Descriptions.

$AB := 1 \quad AN := 3 \qquad BN := AN - AB \qquad CE := \dfrac{BN}{2}$

$AC := AB + CE \qquad AE := AC \qquad AF := \dfrac{2 \cdot AE^2 - CE^2}{2 \cdot AC}$

$CF := AC - AF \qquad CG := \dfrac{BN}{4} \qquad PJ := \sqrt{CG^2 + CF^2}$

$BF := AF - AB \qquad FN := BN - BF \qquad EF := \sqrt{BF \cdot FN} \qquad AH := \sqrt{AC^2 - CG^2}$

$FK := \dfrac{CG \cdot AF}{AH} \qquad EK := EF - FK \qquad EK - \sqrt{FK^2 + CF^2} = 0$

$CP := \dfrac{CE^2}{AC} \qquad 2 \cdot CF - CP = 0$

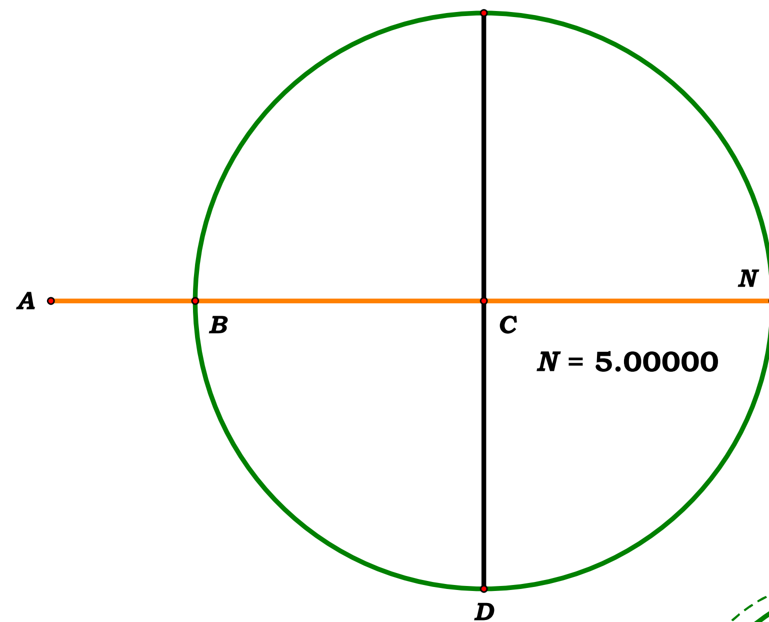
Definitions.

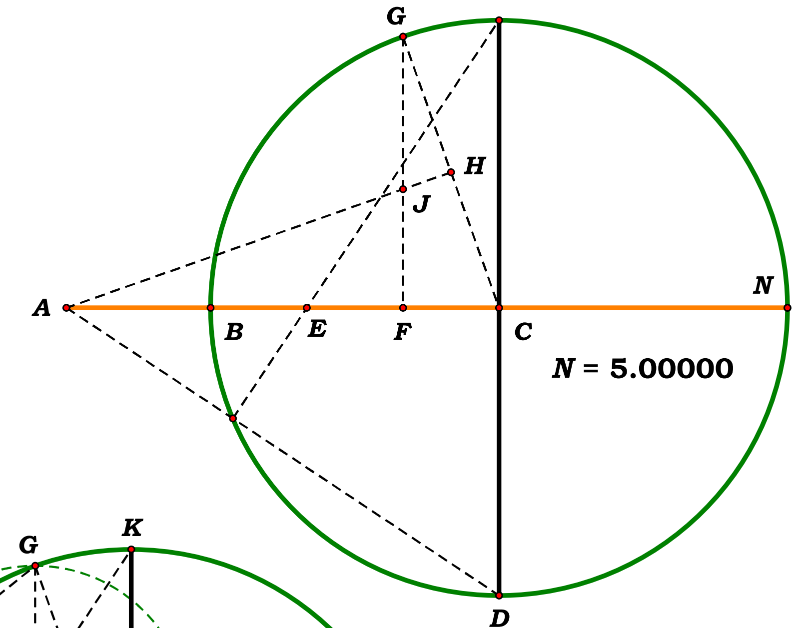
$AB - 1 = 0 \quad AN - AN = 0 \quad BN - (AN - 1) = 0 \quad CE - \dfrac{AN - 1}{2} = 0 \quad AC - \dfrac{AN + 1}{2} = 0 \quad AE - AC = 0$

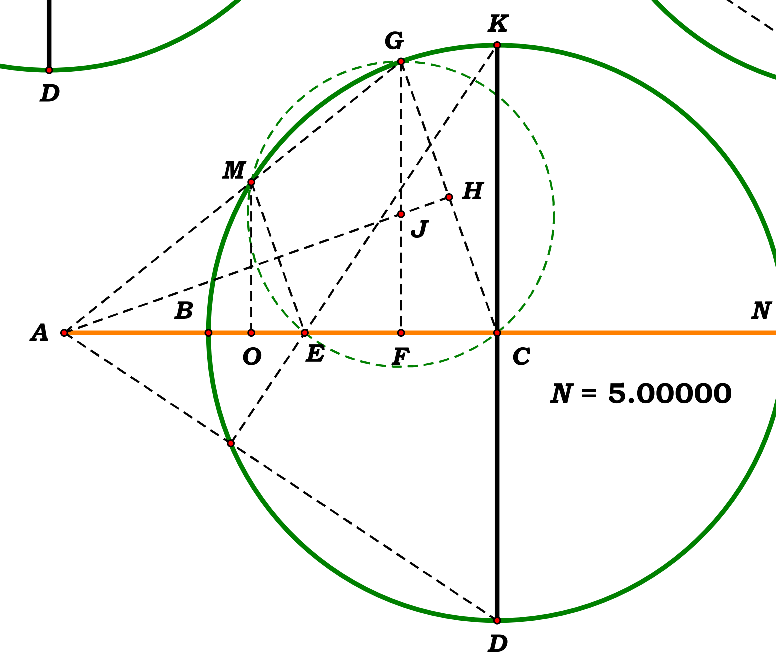
$AF - \dfrac{AN^2 + 6 \cdot AN + 1}{4 \cdot (AN + 1)} = 0 \quad CF - \dfrac{(AN - 1)^2}{4 \cdot (AN + 1)} = 0 \quad CG - \dfrac{BN}{4} = 0 \quad PJ - \sqrt{CG^2 + CF^2} = 0 \quad BF - \dfrac{(AN + 3) \cdot (AN - 1)}{4 \cdot (AN + 1)} = 0$

$FN - \dfrac{(3 \cdot AN + 1) \cdot (AN - 1)}{4 \cdot (AN + 1)} = 0 \quad EF - \dfrac{(AN - 1) \cdot \sqrt{(AN + 3) \cdot (3 \cdot AN + 1)}}{(4 \cdot AN + 4)} = 0 \quad AH - \dfrac{\sqrt{3 \cdot AN^2 + 10 \cdot AN + 3}}{4} = 0 \quad FK - \dfrac{(AN - 1) \cdot \left(AN^2 + 6 \cdot AN + 1\right)}{4 \cdot (AN + 1) \cdot \sqrt{3 \cdot AN^2 + 10 \cdot AN + 3}} = 0$

$EK - \dfrac{AN^2 - 1}{2 \cdot \sqrt{3 \cdot AN^2 + 10 \cdot AN + 3}} = 0 \quad EK - \dfrac{(AN - 1) \cdot (AN + 1)}{2 \cdot \sqrt{3 \cdot AN^2 + 10 \cdot AN + 3}} = 0 \quad CP - \dfrac{(AN - 1)^2}{2 \cdot (AN + 1)} = 0 \quad 2 \cdot CF - CP = 0$







Now, as in one of Superman's Super Villans, get him to say his name backwords.

Descriptions.

$ab := 1 \qquad an := 5 \qquad bn := an - ab \qquad bc := \dfrac{bn}{2}$

$ac := an - bc \qquad ad := \sqrt{ac^2 + bc^2} \qquad ce := \dfrac{bc^2}{ac}$

$cf := \dfrac{ce}{2} \qquad bf := bc - cf \qquad fn := bc + cf \qquad fg := \sqrt{bf \cdot fn}$

$ae := ac - ce \qquad af := ac - cf \qquad ao := \dfrac{af \cdot ae}{ac} \qquad mo := \dfrac{fg \cdot ae}{ac}$

Definitions.

$ab - 1 = 0 \qquad an - an = 0 \qquad bn - (an - 1) = 0 \qquad bc - \dfrac{an - 1}{2} = 0$

$ac - \dfrac{an + 1}{2} = 0 \qquad ad - \dfrac{\sqrt{2} \cdot \sqrt{an^2 + 1}}{2} = 0 \qquad ce - \dfrac{(an - 1)^2}{2 \cdot (an + 1)} = 0$

$cf - \dfrac{(an - 1)^2}{4 \cdot (an + 1)} = 0 \qquad bf - \dfrac{(an + 3) \cdot (an - 1)}{4 \cdot (an + 1)} = 0 \qquad fn - \dfrac{(3 \cdot an + 1) \cdot (an - 1)}{4 \cdot (an + 1)} = 0$

$fg - \dfrac{(an - 1) \cdot \sqrt{(an + 3) \cdot (3 \cdot an + 1)}}{(4 \cdot an + 4)} = 0 \qquad ae - \dfrac{2 \cdot an}{an + 1} = 0 \qquad af - \dfrac{an^2 + 6 \cdot an + 1}{4 \cdot (an + 1)} = 0$

$ao - \dfrac{an \cdot \left(an^2 + 6 \cdot an + 1\right)}{(an + 1)^3} = 0 \qquad mo - \dfrac{an \cdot \sqrt{(an + 3) \cdot (3 \cdot an + 1)} \cdot (an - 1)}{(an + 1)^3} = 0$

$\mathbf{bo} := \mathbf{ao} - \mathbf{ab}$

$\mathbf{on} := \mathbf{bn} - \mathbf{bo}$

$N = 2.17028$

$B = 1.00000$

$BN = 1.17028$

$X = 31.86360$

$Y = 8.78984$

$O = 1.27586$

$BO = 0.27586$

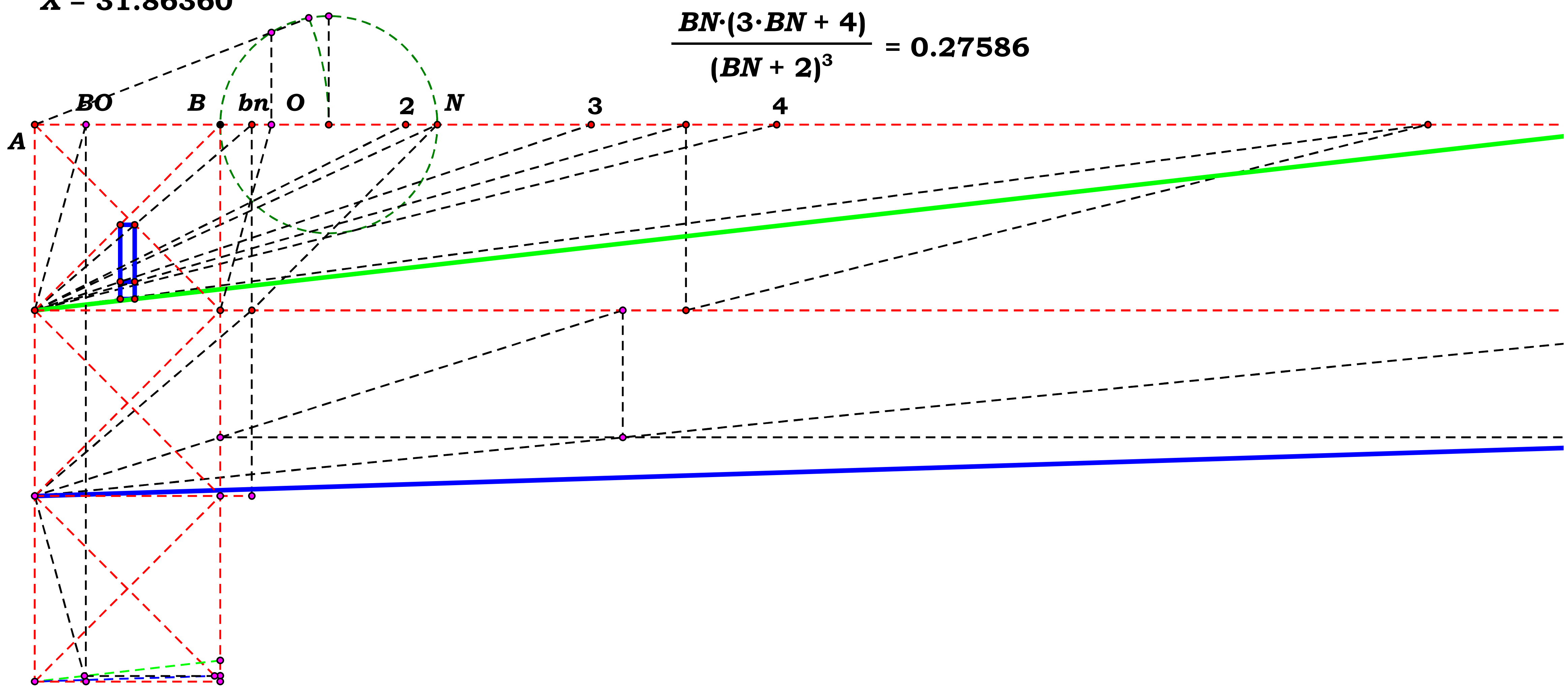
$BN \cdot (3 \cdot BN + 4) = 8.78984$

$(BN + 2)^3 = 31.86360$

$$\frac{BN \cdot (3 \cdot BN + 4)}{(BN + 2)^3} + B = 1.27586$$

$$\frac{BN \cdot (3 \cdot BN + 4)}{(BN + 2)^3} = 0.27586$$

$$\mathbf{bo} - \frac{\mathbf{bn} \cdot (3 \cdot \mathbf{bn} + 4)}{(\mathbf{bn} + 2)^3} = 0$$



$$\mathbf{on} - \frac{\mathbf{bn} \cdot (\mathbf{bn} + 1)^2 \cdot (\mathbf{bn} + 4)}{(\mathbf{bn} + 2)^3} = 0$$

$$\mathbf{bo} - \frac{(\mathbf{an} - 1) \cdot (3 \cdot \mathbf{an} + 1)}{(\mathbf{an} + 1)^3} = 0$$

$$\mathbf{on} - \frac{\mathbf{an}^2 \cdot (\mathbf{an} - 1) \cdot (\mathbf{an} + 3)}{(\mathbf{an} + 1)^3} = 0$$

Taking it easy

Unit.

Given.

Descriptions.

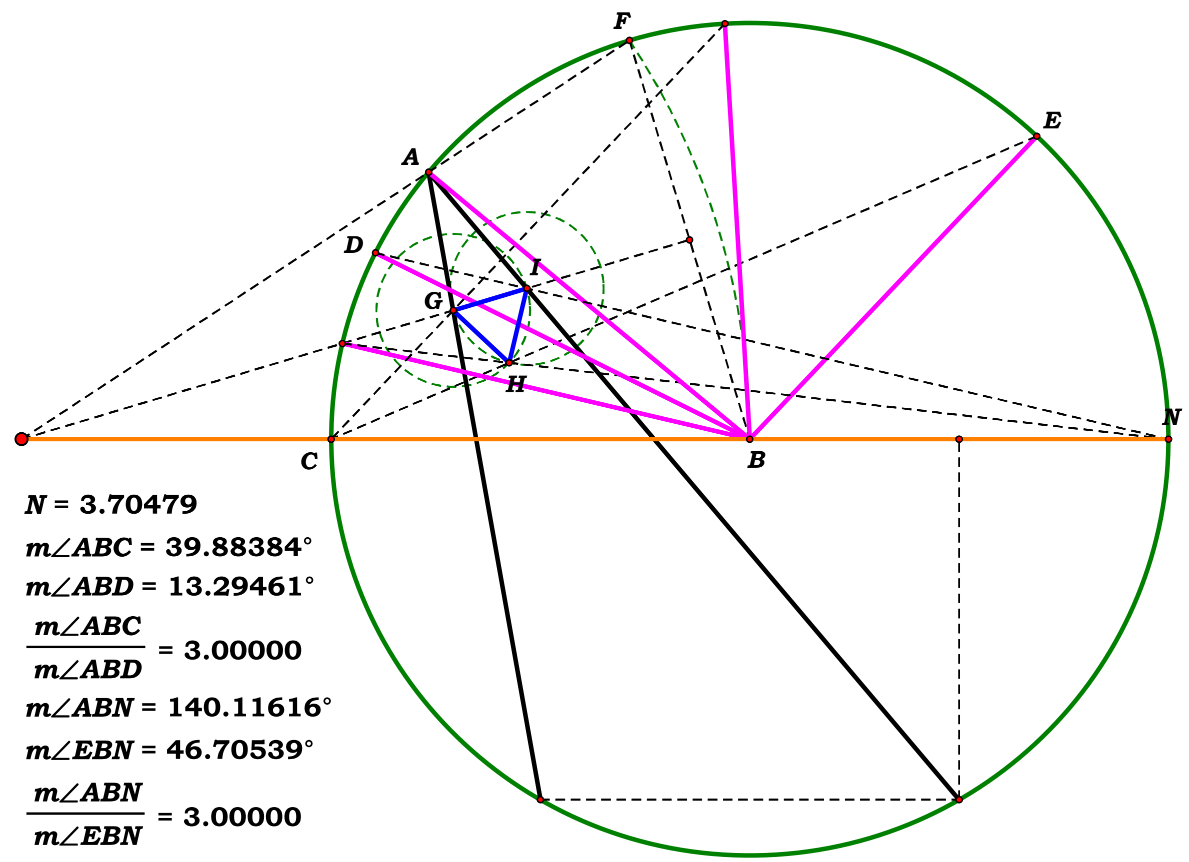
Now, when the brain wants to take a vacation, then any hard work is not on the menu.

Divide CN, BN, BF, by 2 and fill the rest in because basically, it is simpler then the Archimedean Paper Trisector and you always end up with the equilateral triangle in the center.

Not hard to find, not hard to remember.

Definitions.

Coffee and sci-fi.



$N = 3.70479$

$m\angle ABC = 39.88384°$

$m\angle ABD = 13.29461°$

$\dfrac{m\angle ABC}{m\angle ABD} = 3.00000$

$m\angle ABN = 140.11616°$

$m\angle EBN = 46.70539°$

$\dfrac{m\angle ABN}{m\angle EBN} = 3.00000$

$m\angle GHI = 60.00000°$

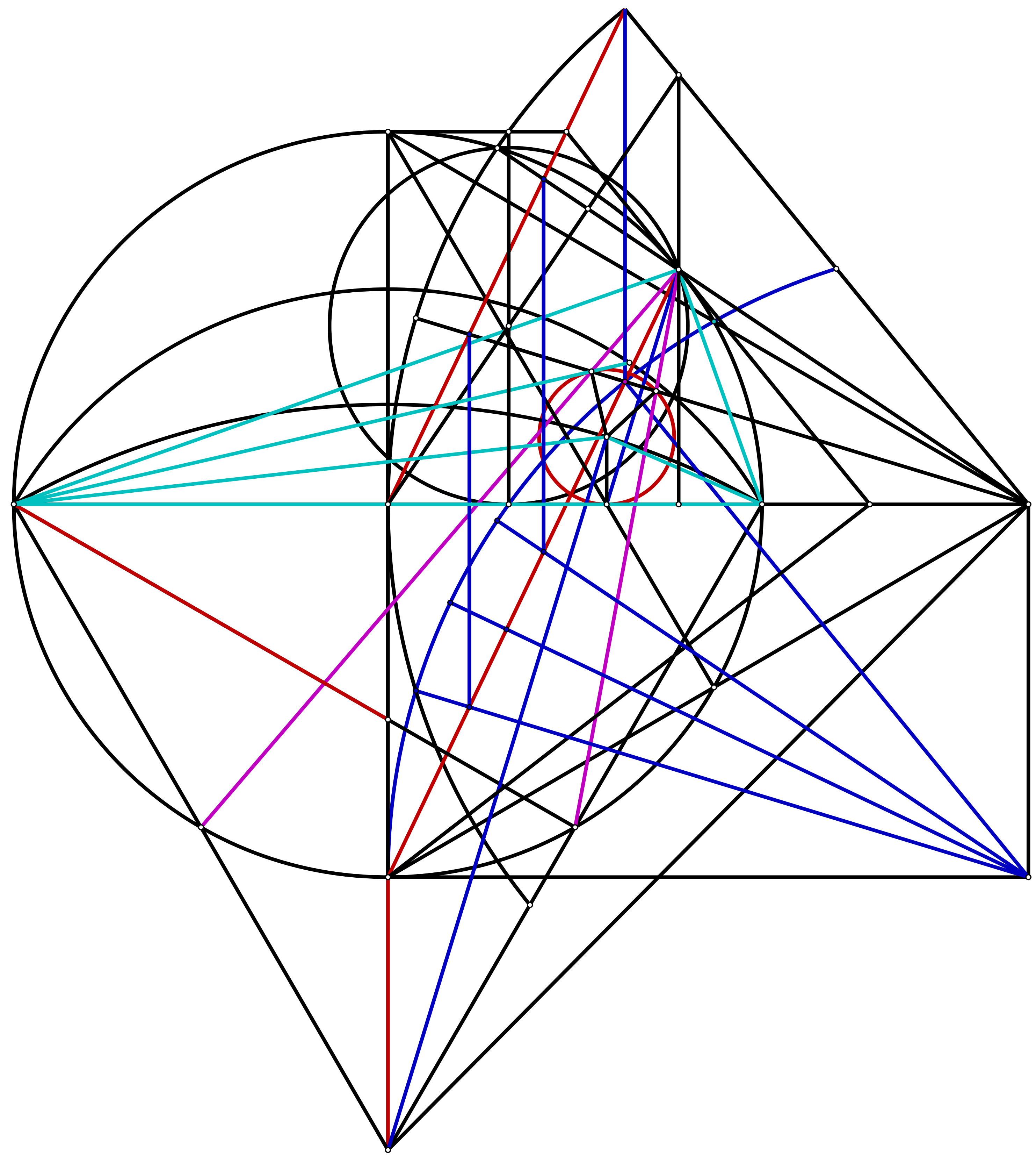
Unit.
Given.

Wall Paper

Descriptions.

Picking mushrooms.

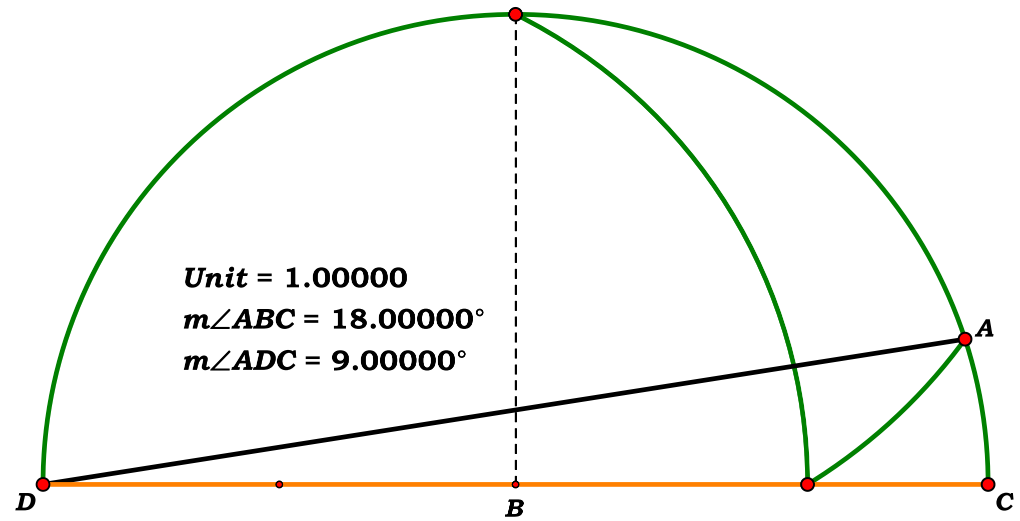
Definitions.



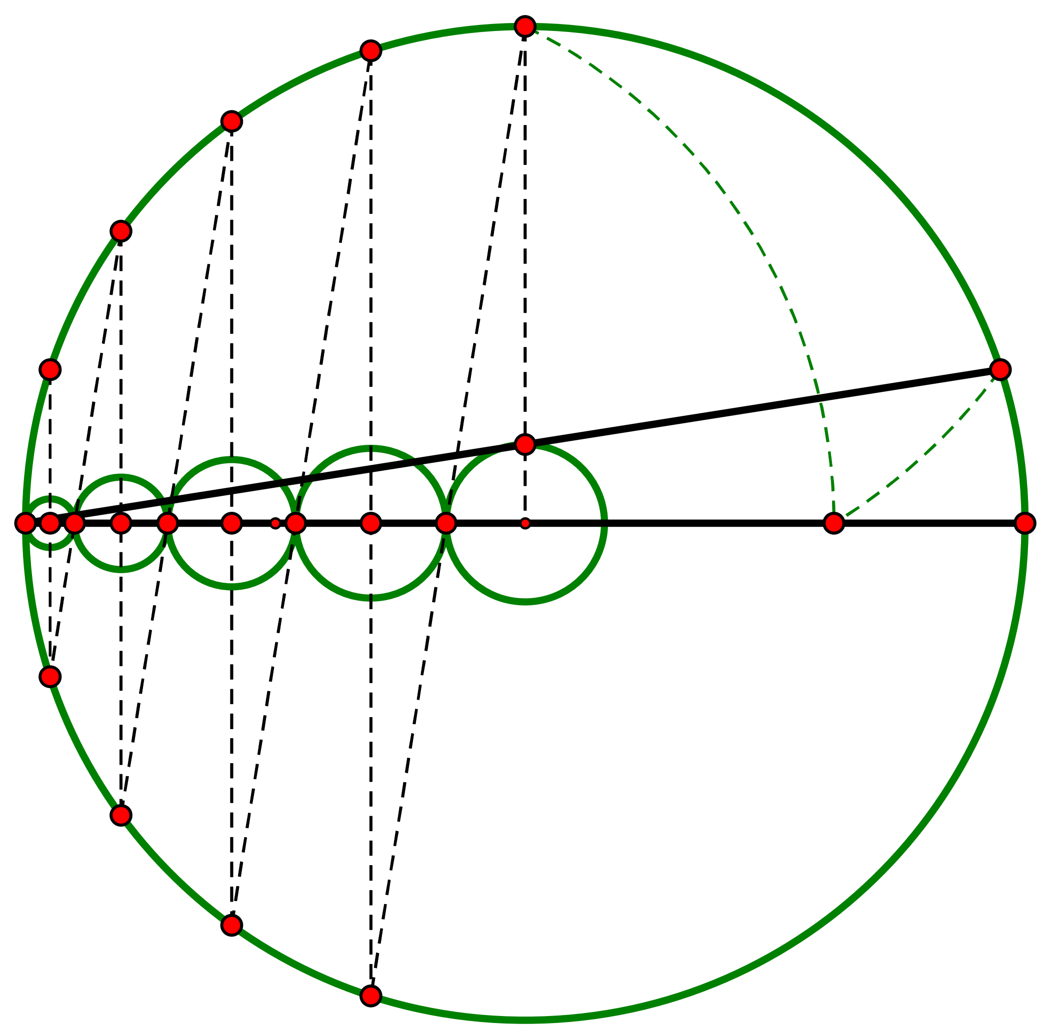
Eliptic Progression or
Baby it is cold outside.

Descriptions.

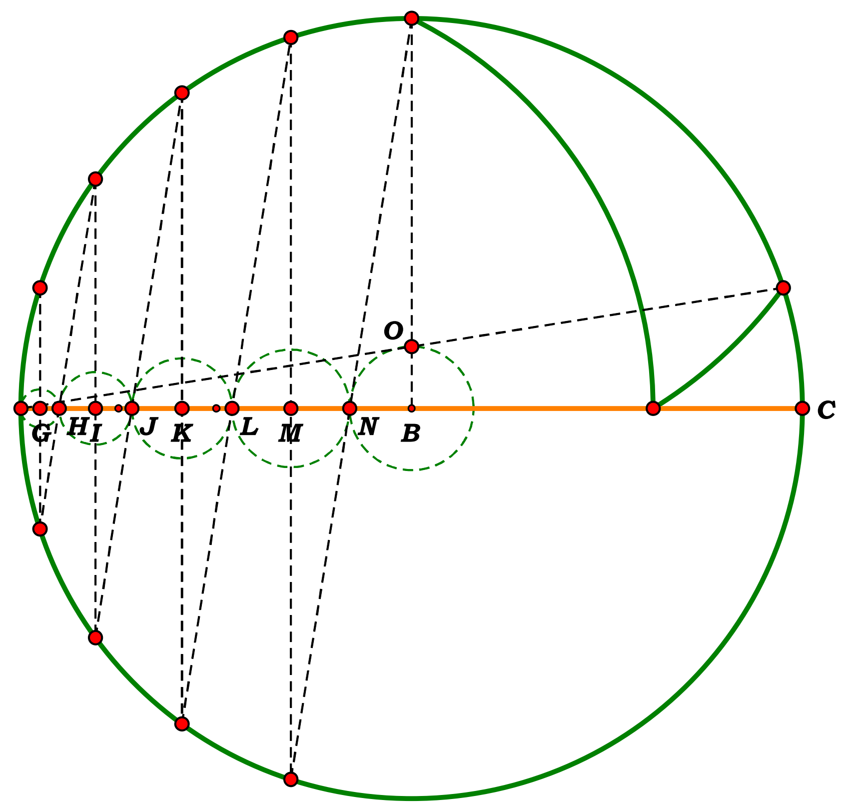
A little review on how to draw a pentagon which will be needed for our weather report.



I can straight forward use the figure to divide the whole circle into twenty parts. This is the proportional method, instead of drawing twenty individual circles aithmetically.



The area of each circle has a specific mathematical relationship.



Area $\odot GH$ = 0.29403 cm^2
Area $\odot IJ$ = 1.06380 cm^2
Area $\odot KL$ = 2.01529 cm^2
Area $\odot MN$ = 2.78506 cm^2
Area $\odot BO$ = 3.07909 cm^2

(Area $\odot GH$) + (Area $\odot MN$) − (Area $\odot BO$) = 0.00000 cm^2
(Area $\odot IJ$) + (Area $\odot KL$) − (Area $\odot BO$) = 0.00000 cm^2

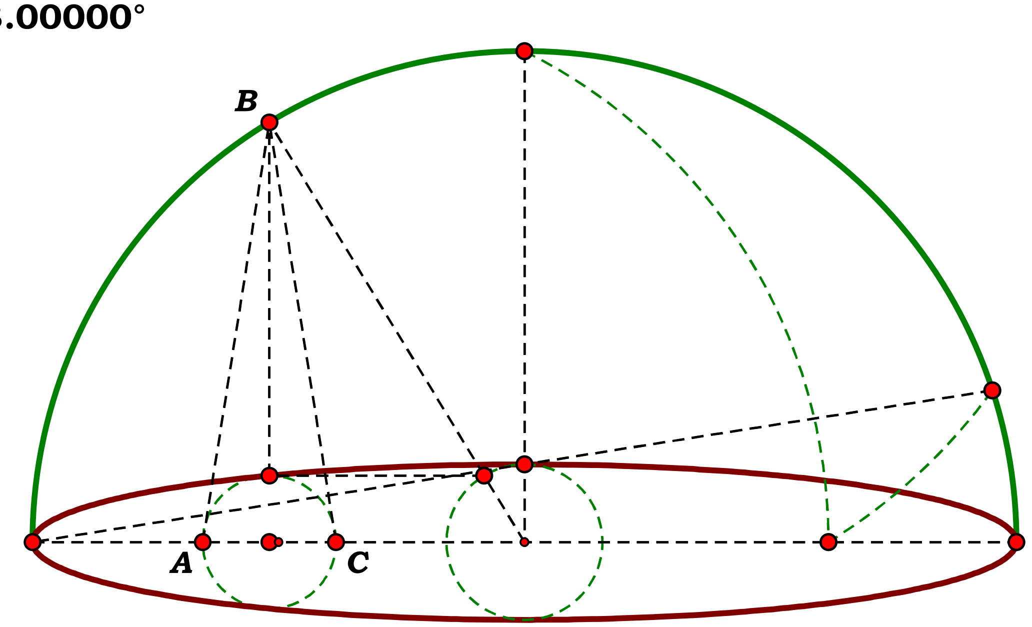
$$\frac{\text{Area } \odot MN}{\text{Area } \odot GH} = 9.47214$$

$$\frac{\text{Area } \odot KL}{\text{Area } \odot IJ} = 1.89443$$

$$\frac{\left(\dfrac{\text{Area } \odot MN}{\text{Area } \odot GH}\right)}{\left(\dfrac{\text{Area } \odot KL}{\text{Area } \odot IJ}\right)} = 5.00000$$

$m\angle ABC$ **= 18.00000°**

That relationsip is elliptical so that no matter where B is and AC has enough line to construct AC, it will always be the same angle as the circle on the minor axis.

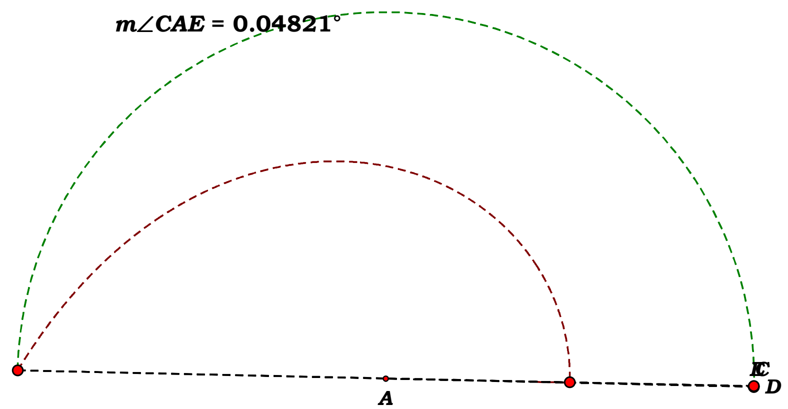


The totle number of circles will always be the number of parts that the quarter circle will be divided.

$\frac{m\angle CAD}{m\angle CAE} = 3.00000$

$m\angle CAD = 0.14462°$

$m\angle CAE = 0.04821°$



Unit = 1.00000

Area ⊙*AB* = 0.07516 cm²

Area ⊙*CD* = 0.28920 cm²

Area ⊙*EF* = 0.60953 cm²

Area ⊙*GH* = 0.98738 cm²

Area ⊙*IJ* = 1.36524 cm²

Area ⊙*KL* = 1.68557 cm²

Area ⊙*MN* = 1.89961 cm²

Area ⊙*OP* = 1.97477 cm²

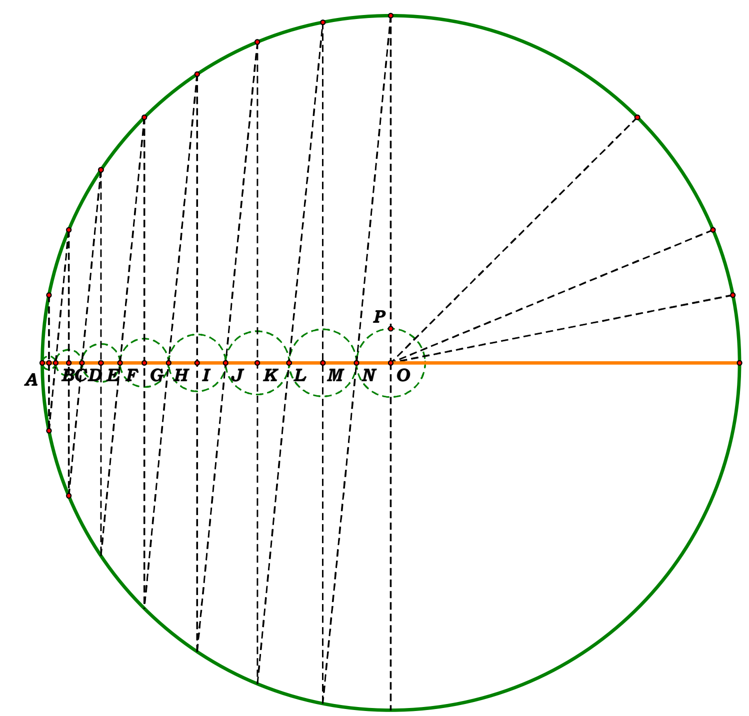
(Area ⊙*AB*) + (Area ⊙*MN*) = 1.97477 cm²

(Area ⊙*CD*) + (Area ⊙*KL*) = 1.97477 cm²

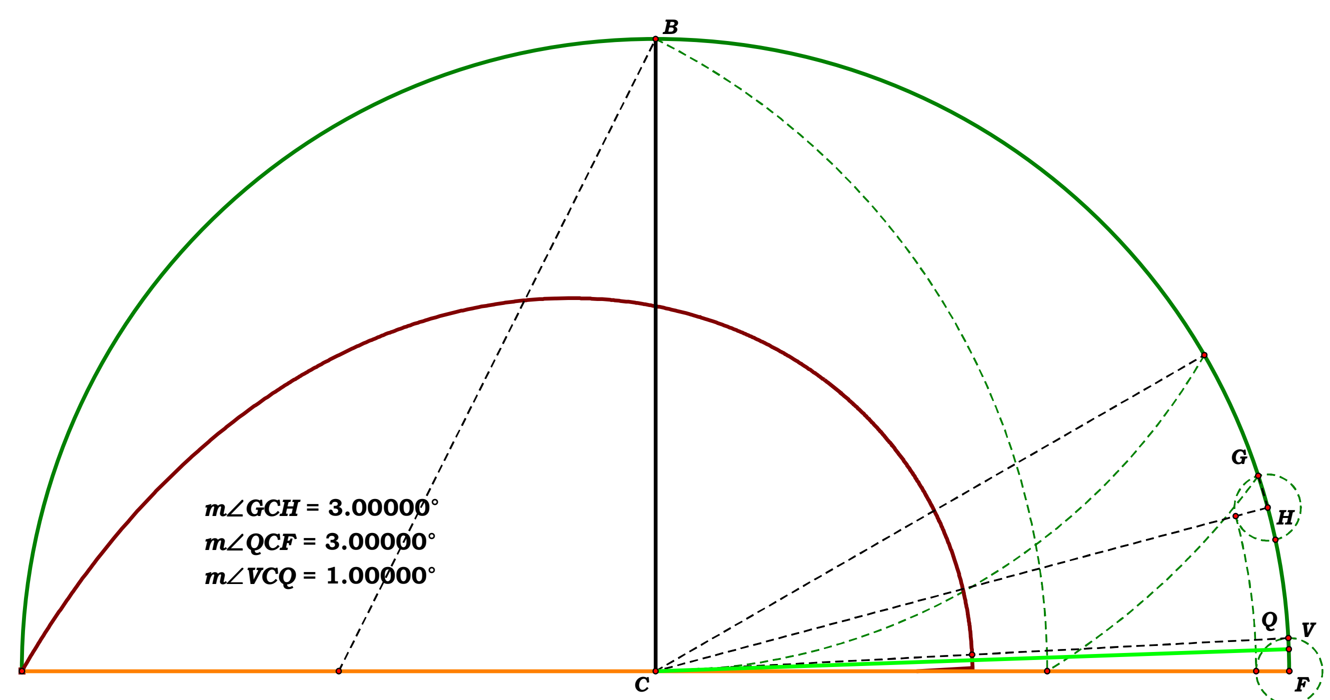
(Area ⊙*EF*) + (Area ⊙*IJ*) = 1.97477 cm²

2·(Area ⊙*GH*) = 1.97477 cm²

$\frac{2\cdot(\text{Area } \odot GH)}{\text{Area } \odot GH} = 2.00000$



So that the difference between 15 and 18 gives 3, which when trisected gives us 1 degree by which one can, if they have nothing better to do, draw away at it to keep them warm and fuzzy.
I made me a little Total Recall tool so I can sleep in. I wonder how big a screen I would need to see my tools angle?



Blast from the Past

Descriptions.

$AB := 1 \quad N := 5 \quad AF := AB \cdot N \quad BF := AF - AB$

$BE := \dfrac{BF}{2} \quad EK := BE \quad AE := AB + BE \quad DE := \dfrac{EK^2}{AE}$

$EF := BE \quad FM := BF \quad EM := \sqrt{FM^2 - EF^2} \quad GM := FM$

$GQ := DE \quad MQ := \sqrt{GM^2 - GQ^2} \quad EQ := MQ - EM$

$DG := EQ \quad EQ = 0.307135$

Definitions.

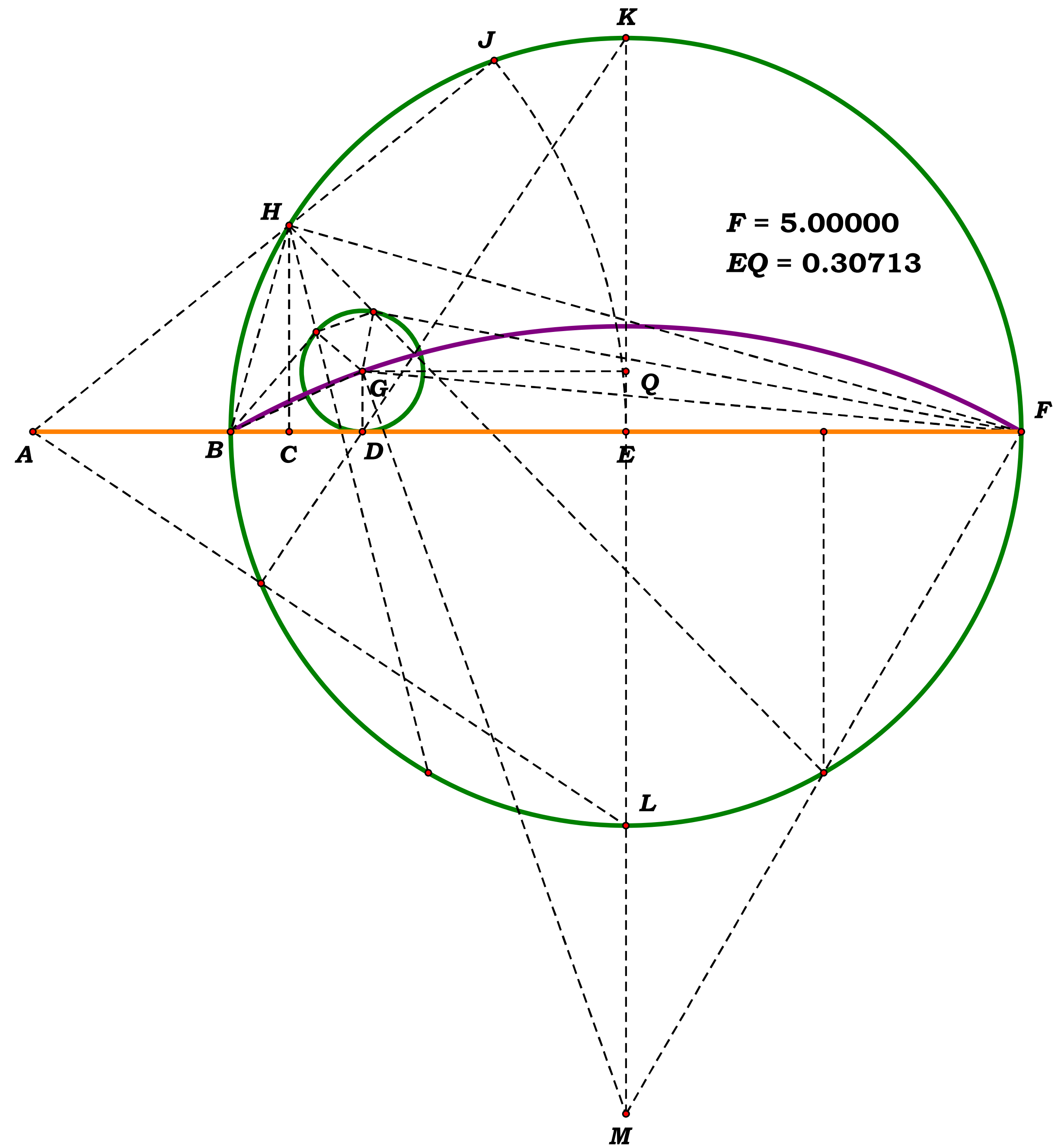
$AB - 1 = 0 \quad N - N = 0 \quad AF - N = 0 \quad BF - (N-1) = 0$

$BE - \dfrac{N-1}{2} = 0 \quad EK - BE = 0 \quad AE - \left(\dfrac{N+1}{2}\right) = 0$

$DE - \dfrac{(N-1)^2}{2 \cdot (N+1)} = 0 \quad EF - BE = 0 \quad FM - BF = 0$

$EM - \dfrac{\sqrt{3} \cdot (N-1)}{2} = 0 \quad GM - FM = 0 \quad GQ - DE = 0 \quad MQ - \dfrac{\sqrt{(N+3) \cdot (3 \cdot N + 1)} \cdot (N-1)}{2 \cdot (N+1)} = 0$

$EQ - \dfrac{(N-1) \cdot \left[\sqrt{[(N+3) \cdot (3 \cdot N + 1)]} - \sqrt{3} \cdot N - \sqrt{3}\right]}{2 \cdot (N+1)} = 0 \quad DG - EQ = 0 \quad EQ = 0.307135$



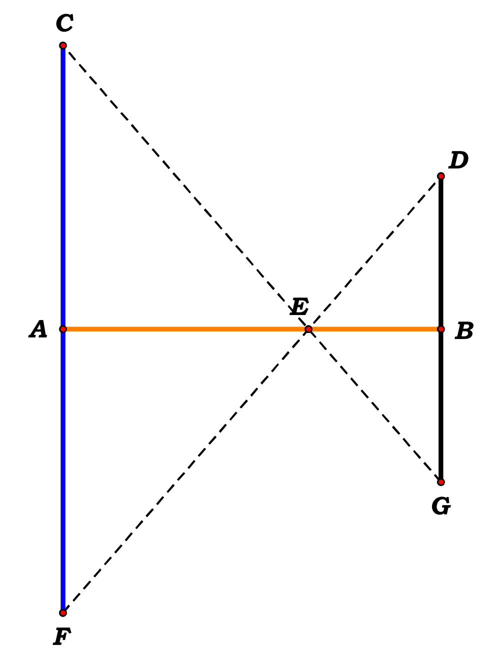
Treasure Island

Cartography Review

The missing treasure is at X, find its coordinates

One of the things writing up dynamic figures does, is make one sloppy when it comes to static figures, one habitually tries to start off with a variable, which is not actually permitted unless it is a study plate to help learn where to find the static for input.
So, in order to help me get out of a bad habit, I decided to take a vacation to Treasure Island. Once arriving I found out that in order to leave the Island, I had to find the treasure maked on the map. It held a first class ticket home.

A = 0.00000
B = 1.00000
C = 0.75346
D = 0.40636
E = 0.64964
H = 1.53933



AB = 6.25062 cm
AC = 4.70958 cm
BD = 2.54000 cm
AE = 4.06063 cm
AH = 9.62175 cm

Descriptions.

$$\mathbf{AB := 1 \quad AC := .75346 \quad BD := .40636}$$

$$\mathbf{BC := \sqrt{AB^2 + AC^2} \qquad BH := AB \cdot \frac{BD}{AC} \qquad AH := AB + BH}$$

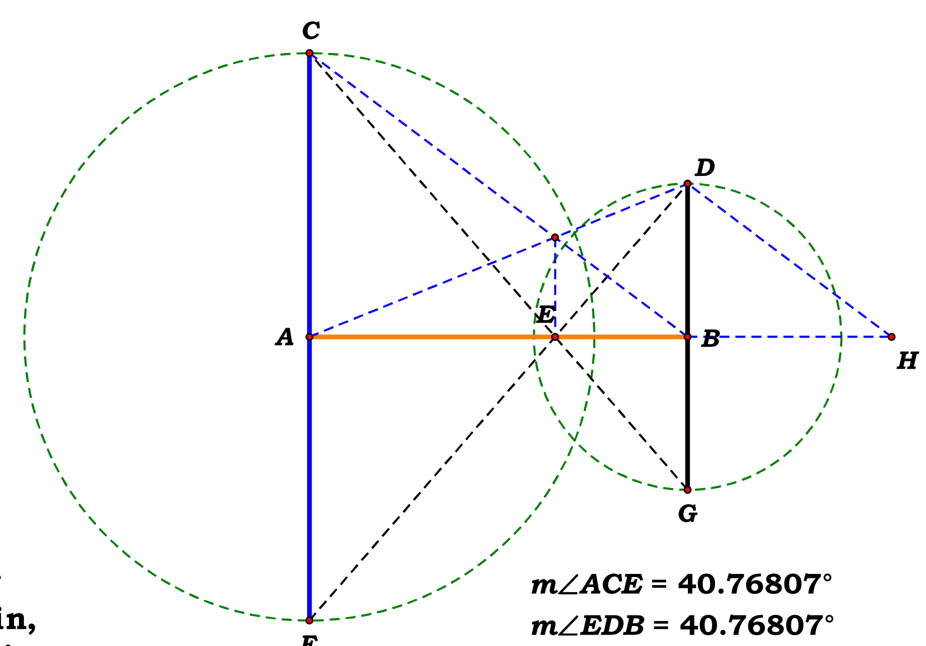
$$\mathbf{BE := \frac{BH \cdot AB}{AH} \qquad AE := AB - BE}$$

Definitions.

$$\mathbf{AB := AB \qquad AC := AC \qquad BD := BD \qquad BC - \sqrt{AB^2 + AC^2} = 0}$$

$$\mathbf{BH - AB \cdot \frac{BD}{AC} = 0 \qquad AH - \frac{AB \cdot (AC + BD)}{AC} = 0}$$

$$\mathbf{BE - \frac{AB \cdot BD}{AC + BD} = 0 \qquad AE - \frac{AB \cdot AC}{AC + BD} = 0}$$



$m\angle ACE$ = 40.76807°
$m\angle EDB$ = 40.76807°

This may be helpful in angle writeups as X always sets the angle, or the proportion. As written by Euclid, angle is an inclination, or again, propotion. Therefore, contrary to current dogma, proportion starts in Book 1. Angle and ratio are synonyms.

The Geometric Unit

The geometric circle affords us two distinct triangles to work with, one using the diameter, and the other the radius and therefore, allows us to convert the Arithmetic circle, into the Geometric one which is determined by the radius.

Descriptions.

$AC := 1 \quad BC := \dfrac{AC}{2} \quad CD := \sqrt{\dfrac{2}{3}} \qquad AD := \sqrt{\dfrac{1}{3}}$

$S_1 := AC \qquad S_2 := AD \quad S_3 := CD$

$$AE := \frac{AC^2 + AD^2 - CD^2}{2 \cdot AC} \qquad AE = 0.333333 \qquad AD^2 = 0.333333$$

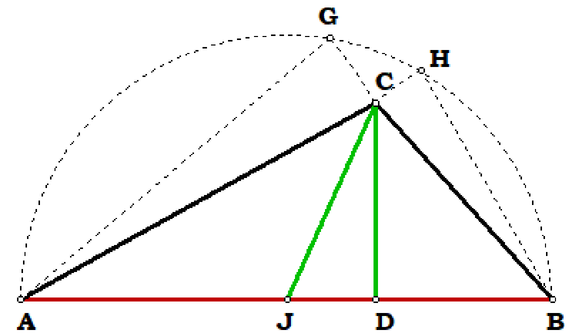
$$CE := AC - AE \qquad DE := \sqrt{AE \cdot (1 - AE)}$$

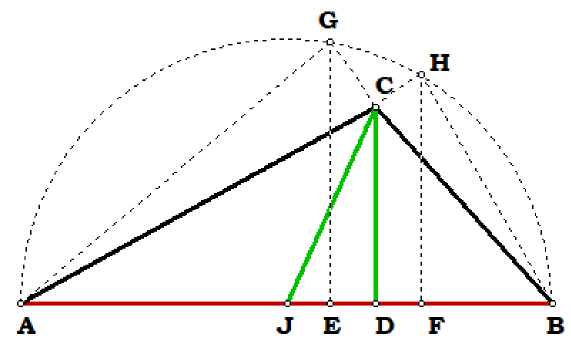
Definitions.

$$AE - \left(1 - CD^2\right) = 0 \qquad CE - CD^2 = 0$$

$$DE - \sqrt{\left(CD^2 - CD^4\right)} = 0$$

$$AD - \sqrt{1 - CD^2} = 0 \qquad \frac{1}{3} + \frac{2}{3} = 1$$





Pythagoras Revisited for the Circle

$S_1 := AB$

$S_2 := AC$

$S_3 := BC$

$$EF - \frac{S_1^2 - S_2^2 - S_3^2}{S_1} = 0 \qquad DE - \frac{S_1^2 - S_2^2 - S_3^2}{2 \cdot S_1} = 0$$

$$AD - \frac{S_1^2 + S_2^2 - S_3^2}{2 \cdot S_1} = 0 \qquad BD - \frac{S_1^2 - S_2^2 + S_3^2}{2 \cdot S_1} = 0$$

$$DJ - \frac{\sqrt{\left(S_2^2 - S_3^2\right)^2}}{2 \cdot S_1} = 0 \qquad CJ - \frac{\sqrt{2 \cdot S_2^2 - S_1^2 + 2 \cdot S_3^2}}{2} = 0$$

$$CD - \frac{\sqrt{\left[\left(S_1 + S_2 - S_3\right) \cdot \left(S_1 - S_2 + S_3\right) \cdot \left(S_2 - S_1 + S_3\right) \cdot \left(S_1 + S_2 + S_3\right)\right]}}{2 \cdot S_1} = 0$$

$A = 0.00000$

$AE = 0.33333$

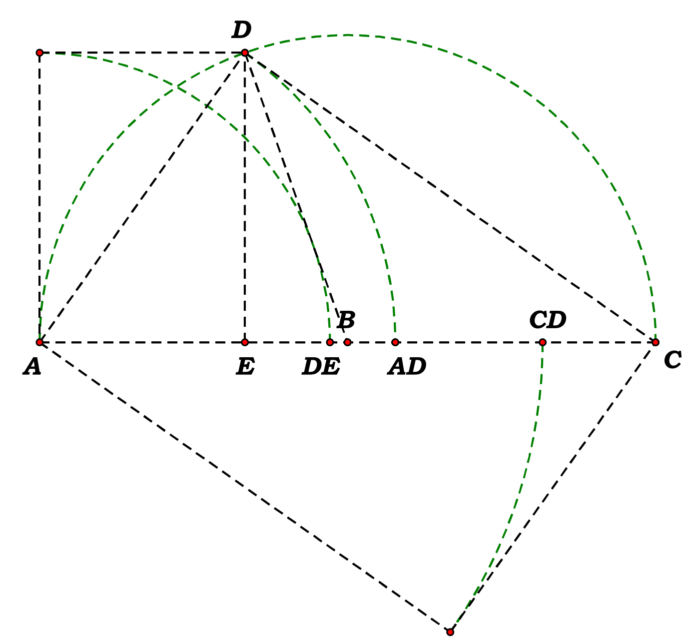
$DE = 0.47140$

$B = 0.50000$

$AD = 0.57735$

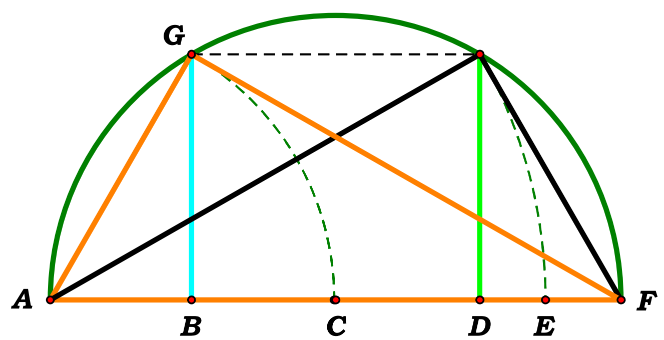
$CD = 0.81650$

$C = 1.00000$



Where did the pie go?

$A = 0.00000$
$B = 0.24744$
$C = 0.49743$
$D = 0.75256$
$E = 0.86750$
$F = 1.00000$
$G = 0.43152$

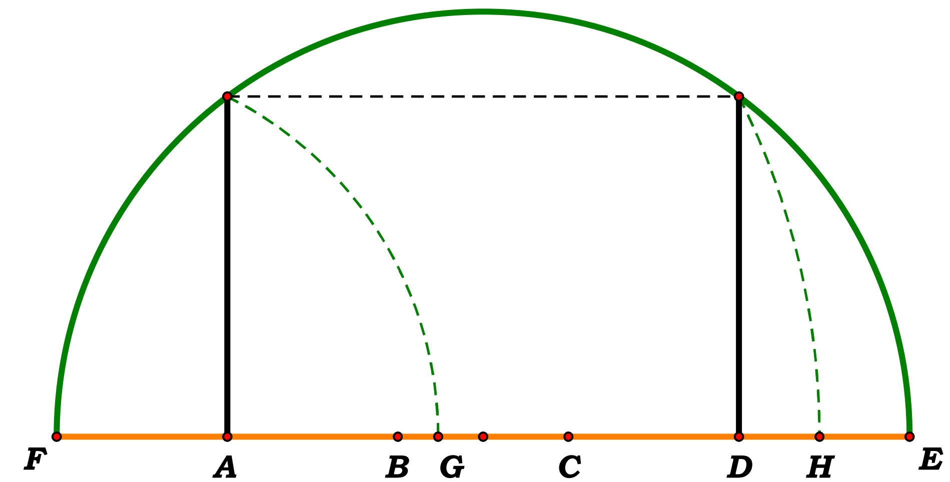


$AF = 8.34083$ cm
$AB = 2.06386$ cm
$AD = 6.27697$ cm
$AC = 4.14901$ cm
$AE = 7.23569$ cm

$B + D = 1.00000$ $\quad (AB + AD) - AF = 0.00000$ cm

$C^2 + E^2 = 1.00000$ $\quad \dfrac{AC^2 + AE^2}{AF^2} = 1.00000$

Which is according to the sum of the squares etc., of Pythagoras translated into simple Arithmetic. And as Pythagoras Revisited would show, all of triangular equatios resolve to arithmetic expressions of proportion, exactly as the angle is defined in Book 1 of the elements as an inclination, i.e., proportion. One will notice, π has noting whatsoever to do with it.



$A = 0.20000$ $\quad \dfrac{1}{A} = 5.00000$

$B = 0.40000$ $\quad \dfrac{2}{B} = 5.00000$

$C = 0.60000$ $\quad \dfrac{3}{C} = 5.00000$

$D = 0.80000$ $\quad \dfrac{4}{D} = 5.00000$

$E = 1.00000$ $\quad \dfrac{5}{E} = 5.00000$

$G = 0.44721$ $\quad G^2 = 0.20000$
$H = 0.89443$ $\quad H^2 = 0.80000$

$A + D = 1.00000$
$G^2 + H^2 = 1.00000$

Ellipse 2

Descriptions.

$$AC := 1 \qquad AB := \frac{AC}{2} \qquad AL := \frac{1}{\sqrt{2}}$$

$$BL := AL - AB \qquad AE := AB - BL \qquad AD := \frac{AE}{2}$$

$$CD := AC - AD \qquad DF := \sqrt{AD \cdot CD} \qquad FH := 2 \cdot DF$$

$$EH := \sqrt{DF^2 + AD^2} \qquad EG := \sqrt{AB^2 + BL^2}$$

$$GH := EH + EG$$

$$2 \cdot FH^2 = 1$$

Definitions.

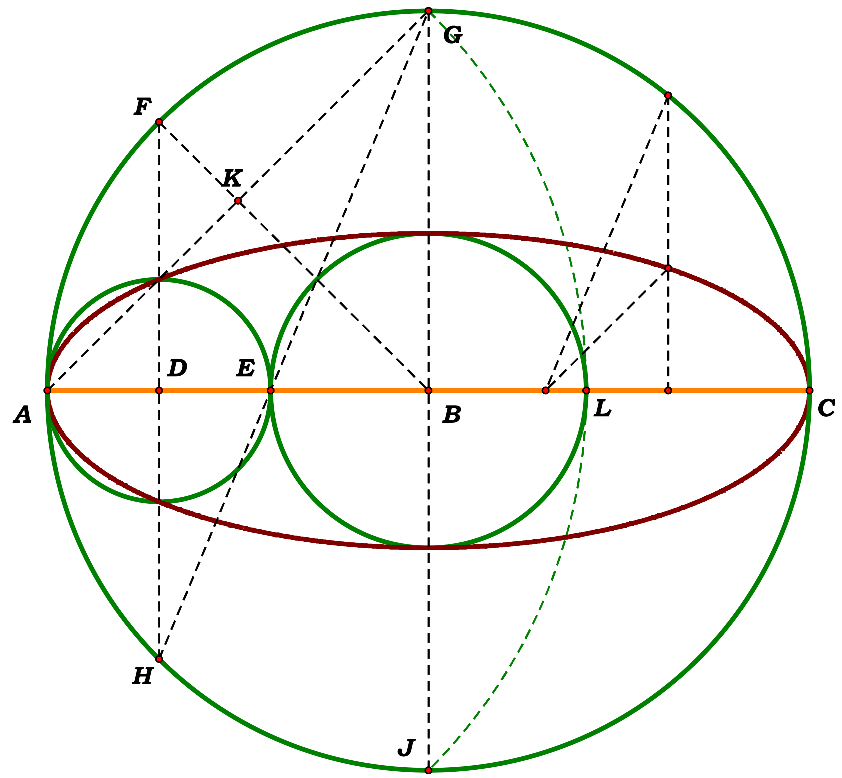
$$AC - 1 = 0 \qquad AB - \frac{1}{2} = 0 \qquad AL - \frac{1}{\sqrt{2}} = 0 \qquad BL - \frac{\sqrt{2}-1}{2} = 0$$

$$AE - \frac{2-\sqrt{2}}{2} = 0 \qquad AD - \frac{2-\sqrt{2}}{4} = 0 \qquad CD - \frac{\sqrt{2}+2}{4} = 0$$

$$DF - \frac{\sqrt{2}}{4} = 0 \qquad FH - \frac{\sqrt{2}}{2} = 0 \qquad EH - \frac{\sqrt{2-\sqrt{2}}}{2} = 0 \qquad EG - \left(\frac{\sqrt{2-\sqrt{2}}}{\sqrt{2}}\right) = 0$$

$$GH - \frac{\left(\sqrt{2}+1\right) \cdot \sqrt{2-\sqrt{2}}}{2} = 0$$

A = 0.00000
D = 0.14645
E = 0.29289
B = 0.50000
L = 0.70711
C = 1.00000



Ellipse 3

Descriptions.

$AC := 1 \quad AB := \dfrac{AC}{2} \quad AF := \dfrac{AB}{2} \quad CF := AC - AF$

$BF := AB - AF \quad DK := BF \quad FL := \sqrt{AF \cdot CF} \quad AD := AB - FL$

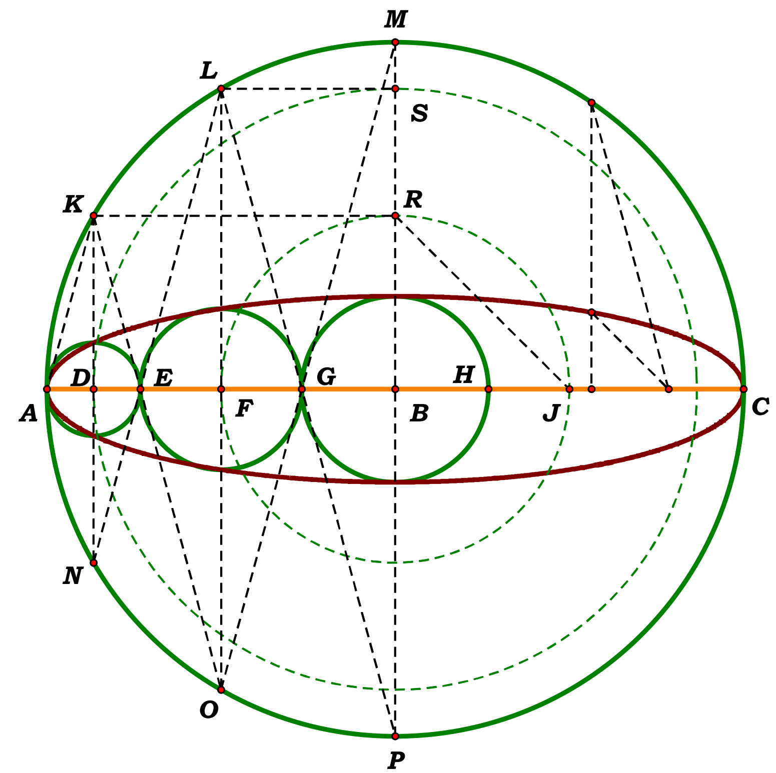
$AE := 2 \cdot AD \quad DK := BF \quad EF := AF - AE \quad AG := AF + EF$

$BG := AB - AG \quad AK := \sqrt{DK^2 + AD^2} \quad KN := 2 \cdot DK$

$LO := 2 \cdot FL$

$LO^2 + KN^2 = 1$

$A = 0.00000$
$D = 0.06699$
$E = 0.13397$
$F = 0.25000$
$G = 0.36603$
$B = 0.50000$
$H = 0.63397$
$J = 0.75000$
$H = 0.63397$
$J = 0.75000$
$C = 1.00000$
$D = 0.25000$
$L = 0.43301$



Definitions.

$AC - 1 = 0 \quad AB - \dfrac{1}{2} = 0 \quad AF - \dfrac{1}{4} = 0 \quad CF - \dfrac{3}{4} = 0 \quad BF - \dfrac{1}{4} = 0 \quad DK - BF = 0$

$FL - \dfrac{\sqrt{3}}{2^2} = 0 \quad AD - \dfrac{2 - \sqrt{3}}{4} = 0 \quad AE - \dfrac{2 - \sqrt{3}}{2} = 0 \quad DK - BF = 0 \quad EF - \dfrac{2 \cdot \sqrt{3} - 3}{4} = 0$

$AG - \dfrac{2 \cdot \sqrt{3} - 2}{4} = 0 \quad BG - \dfrac{2 - \sqrt{3}}{2} = 0 \quad AK - \dfrac{\sqrt{2 - \sqrt{3}}}{2} = 0 \quad KN - \dfrac{1}{2} = 0 \quad LO - \dfrac{\sqrt{3}}{2} = 0$

Ellipse 4

Descriptions.

$AC := 1 \quad AB := \frac{AC}{2} \quad AR := \sqrt{2 \cdot AB^2} \quad AY := \frac{AR}{2} \quad BY := AY$

$OY := AB - BY \quad AF := OY \quad FO := BY \quad AO := \sqrt{FO^2 + AF^2}$

$AW := \frac{AO}{2} \quad BW := \sqrt{AB^2 - AW^2} \quad NW := AB - BW \quad AD := NW$

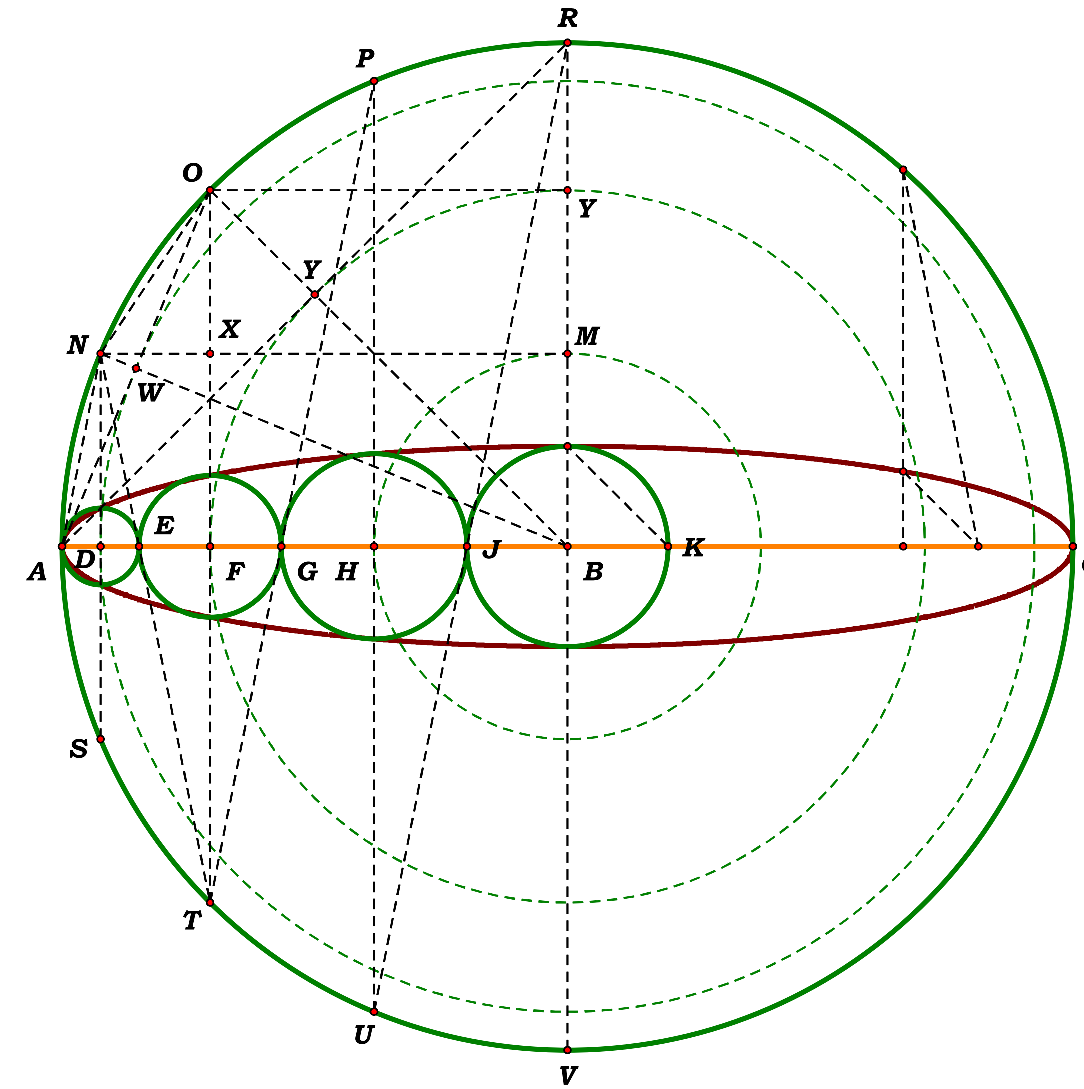
$CD := AC - AD \quad DN := \sqrt{AD \cdot CD} \quad AE := 2 \cdot AD \quad EF := AF - AE$

$AG := AF + EF \quad AH := AB - DN \quad GH := AH - AG \quad AJ := AH + GH$

$BJ := AB - AJ \quad BD := AB - AD \quad HP := BD \quad NS := 2 \cdot DN \quad OT := 2 \cdot FO \quad PU := 2 \cdot HP$

$PU^2 + NS^2 = 1 \quad 2 \cdot OT^2 = 1$

A = 0.00000
D = 0.03806
E = 0.07612
F = 0.14645
G = 0.21677
H = 0.30866
J = 0.40054
B = 0.50000
N = 0.19134
O = 0.35355
P = 0.46194



Definitions.

$AC - 1 = 0 \quad AB - \frac{1}{2} = 0 \quad AR - \frac{\sqrt{2}}{2} = 0 \quad AY - \frac{\sqrt{2}}{4} = 0 \quad BY - AY = 0 \quad OY - \frac{2-\sqrt{2}}{4} = 0 \quad AF - OY = 0 \quad FO - BY = 0$

$AO - \frac{\sqrt{2-\sqrt{2}}}{2} = 0 \quad AW - \frac{\sqrt{2-\sqrt{2}}}{4} = 0 \quad BW - \frac{\sqrt{2+\sqrt{2}}}{2^2} = 0 \quad NW - \frac{2-\sqrt{\sqrt{2}+2}}{4} = 0 \quad AD - NW = 0 \quad CD - \frac{2+\sqrt{\sqrt{2}+2}}{4} = 0 \quad DN - \frac{\sqrt{2-\sqrt{2}}}{2^2} = 0 \quad AE - \frac{2-\sqrt{\sqrt{2}+2}}{2} = 0$

$EF - \frac{2 \cdot \sqrt{\sqrt{2}+2} - \sqrt{2} - 2}{4} = 0 \quad AG - \frac{\sqrt{\sqrt{2}+2} - \sqrt{2}}{2} = 0 \quad AH - \frac{2-\sqrt{2-\sqrt{2}}}{4} = 0 \quad GH - \frac{2 \cdot \left(\sqrt{2} - \sqrt{\sqrt{2}+2} + 1\right) - \sqrt{2-\sqrt{2}}}{4} = 0 \quad AJ - \frac{\sqrt{2} - \sqrt{\sqrt{2}+2} - \sqrt{2-\sqrt{2}} + 2}{2} = 0$

$BJ - \left(\frac{\sqrt{2-\sqrt{2}} + \sqrt{\sqrt{2}+2} - \sqrt{2} - 1}{2}\right) = 0 \quad BD - \frac{\sqrt{\sqrt{2}+2}}{4} = 0 \quad HP - BD = 0 \quad NS - \frac{\sqrt{2-\sqrt{2}}}{2} = 0 \quad OT - \frac{\sqrt{2}}{2} = 0 \quad PU - \frac{\sqrt{\sqrt{2}+2}}{2} = 0$

The 54 degree massacre.

Descriptions.

$$AG := 1 \quad AC := \frac{AG}{2} \quad AB := \frac{AG}{4} \quad BC := AC - AB$$

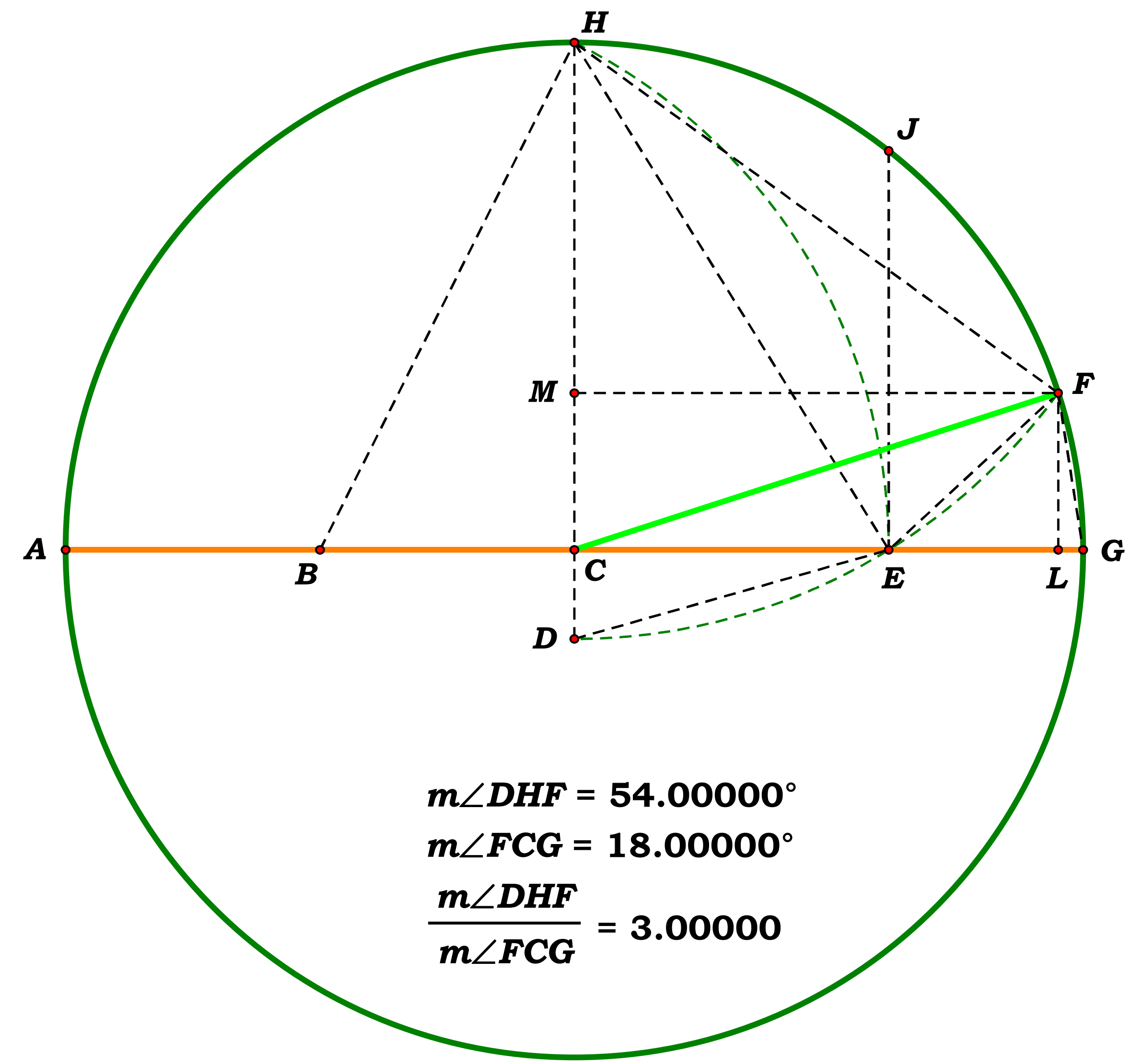
$$BH := \sqrt{BC^2 + AC^2} \quad AE := AB + BH \quad CE := AE - AC$$

$$HE := \sqrt{CE^2 + AC^2} \quad S_1 := AC \quad S_2 := HE \quad S_3 := AC$$

$$HM := \frac{{S_1}^2 + {S_2}^2 - {S_3}^2}{2 \cdot S_1} \quad CM := AC - HM \quad FM := \sqrt{HM \cdot (AG - HM)}$$

$$AL := AC + FM \quad GL := AG - AL \quad FG := \sqrt{CM^2 + GL^2}$$

A = 0.00000
B = 0.25000
C = 0.50000
E = 0.80902
L = 0.97553
G = 1.00000
F = 0.15451
J = 0.39308
M = 0.15451



Definitions.

$$AG - 1 = 0 \quad AC - \frac{1}{2} = 0 \quad AB - \frac{1}{4} = 0 \quad BC - \frac{1}{4} = 0 \quad BH - \frac{\sqrt{5}}{2^2} = 0$$

$$AE - \frac{1+\sqrt{5}}{4} = 0 \quad CE - \frac{\sqrt{5}-1}{4} = 0 \quad HE - \frac{\sqrt{5-\sqrt{5}}}{2^{\frac{3}{2}}} = 0 \quad HM - \frac{5-\sqrt{5}}{8} = 0$$

$$CM - \frac{\sqrt{5}-1}{8} = 0 \quad FM - \frac{\sqrt{5+\sqrt{5}}}{\sqrt{2^5}} = 0 \quad AL - \frac{4+\sqrt{2\cdot\sqrt{5}+10}}{8} = 0 \quad GL - \frac{4-\sqrt{2\cdot\sqrt{5}+10}}{8} = 0 \quad FG - \frac{\sqrt{4-\sqrt{2\cdot\sqrt{5}+10}}}{\sqrt{8}} = 0$$

Ellipse 5

Descriptions.

$AC := 1 \quad AB := \dfrac{AC}{2} \quad AN := \dfrac{4+\sqrt{2\cdot\sqrt{5}+10}}{8} \quad CN := AC - AN$

$MN := CN \quad NS := \sqrt{AN\cdot CN} \quad BF := NS \quad AF := AB + BF$

$CF := AC - AF \quad FO := \sqrt{AF\cdot CF} \quad BE := \dfrac{BF\cdot AB}{AB + FO} \quad AE := AB + BE$

$EF := BF - BE \quad AG := AF + EF \quad BN := AN - AB \quad DS := \sqrt{BN^2 + CF^2}$

$SX := \dfrac{DS}{2} \quad ST := 2\cdot NS \quad PX := \sqrt{ST^2 - SX^2} \quad BX := AB - PX$

$BK := BX \quad AK := AB + BK \quad CK := AC - AK \quad KR := \sqrt{AK\cdot CK}$

$BH := KR \quad CM := 2\cdot CN \quad KM := BK - CM \quad AH := AB + BH$

$GH := AH - AG \quad JH := GH \quad AJ := AH + GH \quad JK := AK - AJ$

$AM := AK + JK \quad CH := AC - AH \quad HP := \sqrt{AH\cdot CH}$

$OW := 2\cdot FO \quad PV := 2\cdot HP \quad RU := 2\cdot KR$

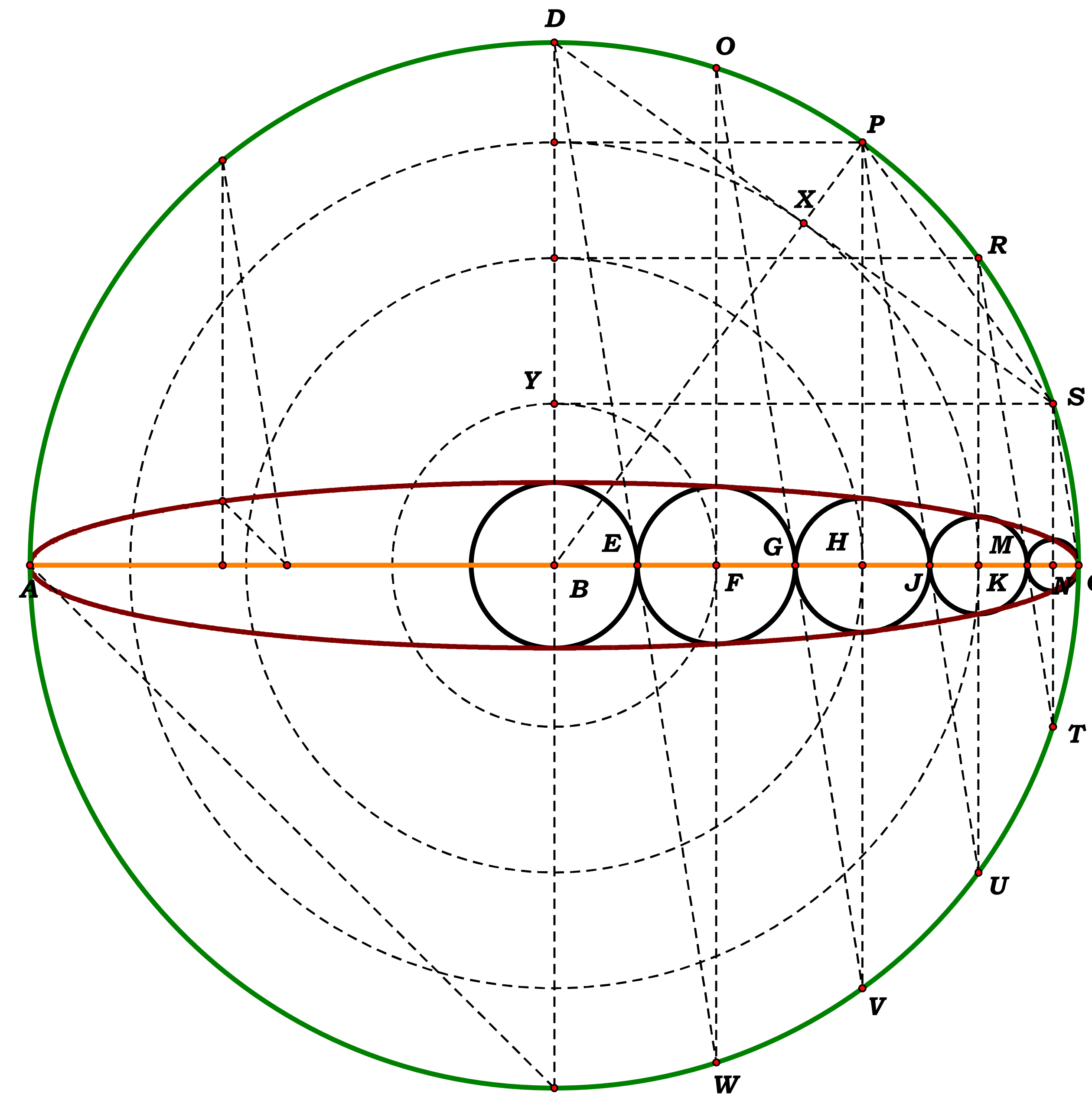
$ST^2 + OW^2 = 1 \quad PV^2 + RU^2 = 1$

Definitions.

$AC - 1 = 0 \quad AB - \dfrac{1}{2} = 0 \quad AN - \dfrac{4+\sqrt{2\cdot\sqrt{5}+10}}{8} = 0 \quad CN - \left(\dfrac{4-\sqrt{2\cdot\sqrt{5}+10}}{8}\right) = 0 \quad MN - CN = 0$

$AE - \dfrac{\sqrt{2\cdot\sqrt{5}+10} - \sqrt{10\cdot\sqrt{5}+50} + 2\cdot\sqrt{5} + 2}{\left(12 - 4\cdot\sqrt{5}\right)} = 0$

$A = 0.00000$
$B = 0.50000$
$E = 0.57919$
$F = 0.65451$
$G = 0.72982$
$H = 0.79389$
$J = 0.85796$
$K = 0.90451$
$M = 0.95106$
$N = 0.97553$
$C = 1.00000$
$O = 0.47553$
$P = 0.40451$
$R = 0.29389$
$S = 0.15451$



$$NS-\frac{\sqrt{3-\sqrt{5}}}{\sqrt{2^5}}=0 \qquad BF-NS=0 \qquad AF-\frac{3+\sqrt{5}}{8}=0$$

$$CF-\frac{5-\sqrt{5}}{8}=0 \qquad FO-\frac{\sqrt{5+\sqrt{5}}}{\sqrt{2^5}}=0 \qquad BE-\frac{\left(\sqrt{5}-1\right)\cdot\left(\sqrt{2\cdot\sqrt{5}+10}-4\right)}{2\cdot\left(2\cdot\sqrt{5}-6\right)}=0$$

Definitions.

$$EF-\frac{\left(\sqrt{5}-1\right)\cdot\left(\sqrt{5}-2\cdot\sqrt{2\cdot\sqrt{5}+10}+5\right)}{8\cdot\left(\sqrt{5}-3\right)}=0 \qquad AG-\frac{\left(\sqrt{5}+1\right)\cdot\left(\sqrt{2\cdot\sqrt{5}+10}-2\right)}{8}=0 \qquad BN-\frac{\sqrt{2\cdot\sqrt{5}+10}}{8}=0$$

$$DS-\frac{\sqrt{5-\sqrt{5}}}{2\cdot\sqrt{2}}=0 \qquad SX-\frac{\sqrt{5-\sqrt{5}}}{4\cdot\sqrt{2}}=0 \qquad ST-\frac{2\cdot\sqrt{3-\sqrt{5}}}{\sqrt{2^5}}=0 \qquad PX-\frac{3-\sqrt{5}}{8}=0 \qquad BX-\frac{1+\sqrt{5}}{8}=0 \qquad BK-BX=0$$

$$AK-\frac{5+\sqrt{5}}{8}=0 \qquad CK-\frac{3-\sqrt{5}}{8}=0 \qquad KR-\frac{\sqrt{5-\sqrt{5}}}{\sqrt{2^5}}=0 \qquad BH-KR=0 \qquad CM-\frac{4-\sqrt{2\cdot\sqrt{5}+10}}{4}=0$$

$$KM-\frac{2\cdot\sqrt{2\cdot\sqrt{5}+10}+\sqrt{5}-7}{8}=0 \qquad AH-\frac{4+\sqrt{2}\cdot\sqrt{5-\sqrt{5}}}{8}=0 \qquad GH-\frac{\sqrt{2}\cdot\sqrt{5-\sqrt{5}}-\sqrt{2\cdot\sqrt{5}+10}-\sqrt{10\cdot\sqrt{5}+50}+2\cdot\sqrt{5}+6}{8}=0$$

$$JH-GH=0 \qquad AJ-\frac{2\cdot\sqrt{2}\cdot\sqrt{5-\sqrt{5}}-\sqrt{2\cdot\sqrt{5}+10}-\sqrt{10\cdot\sqrt{5}+50}+2\cdot\sqrt{5}+10}{8}=0 \qquad JK-\frac{\sqrt{2\cdot\sqrt{5}+10}-2\cdot\sqrt{10-2\cdot\sqrt{5}}+\sqrt{10\cdot\sqrt{5}+50}-\sqrt{5}-5}{8}=0$$

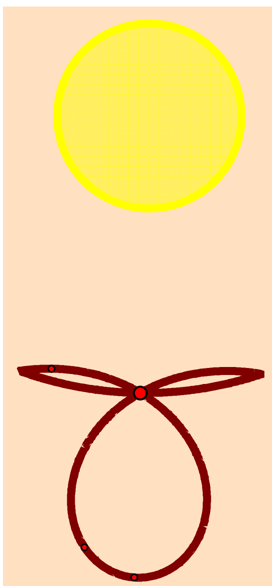
$$AM-\frac{\sqrt{2\cdot\sqrt{5}+10}-2\cdot\sqrt{10-2\cdot\sqrt{5}}+\sqrt{10\cdot\sqrt{5}+50}}{8}=0 \qquad CH-\frac{4-\sqrt{2}\cdot\sqrt{5-\sqrt{5}}}{8}=0 \qquad HP-\frac{1+\sqrt{5}}{8}=0 \qquad OW-\frac{2\cdot\sqrt{5+\sqrt{5}}}{\sqrt{2^5}}=0$$

$$PV-\frac{2\cdot\left(1+\sqrt{5}\right)}{8}=0 \qquad RU-\frac{2\cdot\sqrt{5-\sqrt{5}}}{\sqrt{2^5}}=0$$

The Rabbit Trisector

Saturday, June 17, 2023

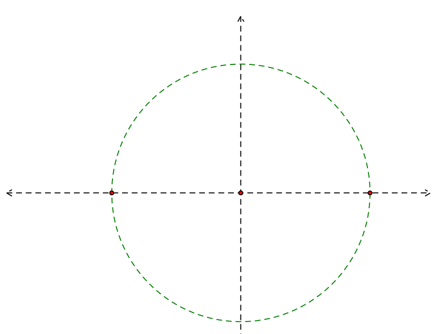
While cleaning out some old plates for Three Pieces of Paper, which I am now working on, I rant (sic) into a single file in a directory for 2006. I also found, that while sitting there, it was not idle, for I found that it had a kid, or rather a little bunny.

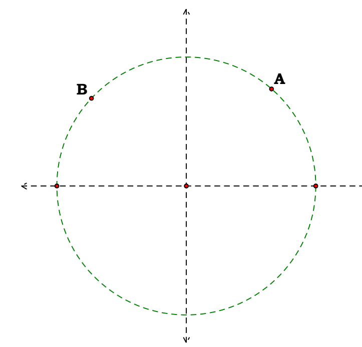


Not being a grown-up bunny, it cannot do anything really, ’cept watch the sun and grow for when it grows up, it will become, by using its ears, a master angle trisector. Hard to believe, I know, but it is a fact which his father will demonstrate for you. His father is rather famous, having made his first movie debut in the film classic, Harvey, which you might want to find a copy of, for in it, you will learn that Harvey is a Pooka, and invisible.

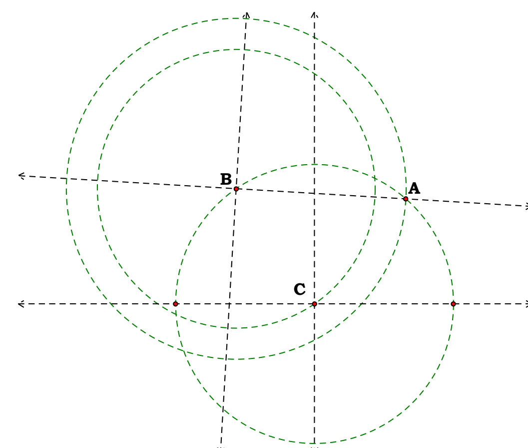
I am, if you have not yet guessed it, an amateur Pooka Hunter. And this is the story of how I found Harvey after he retired from the motion picture industry.

The reason that Harvey was so hard to find is, perhaps you already know, that he is not the product of the Straightedge, or Arithmetic recursion, but of the circle, or Geometric recursion. Therefore, one must learn the magic of the circle in order to peer into the invisible world. I will start there, with the circle and divide it into its quadrants for Harvey can trisect any angle in any quadrant.



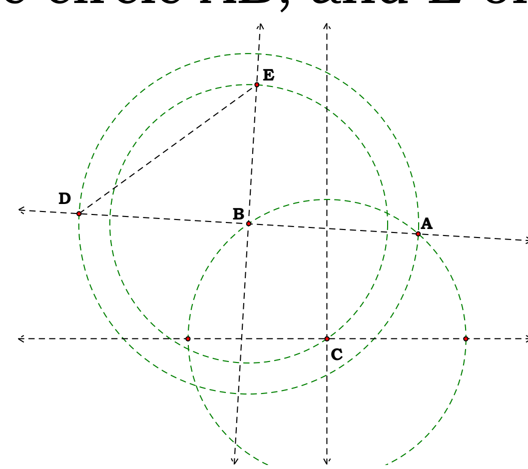


Place upon it, two points, point A for the angle, and B to construct the loci, or Loki, as some have called him, but his name is really Harvey and his origins will be made invisible after we get done outlining him.

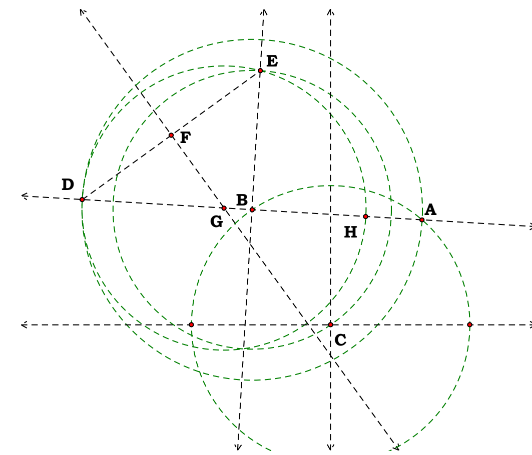


From B, draw two circles, one to A and one to C.

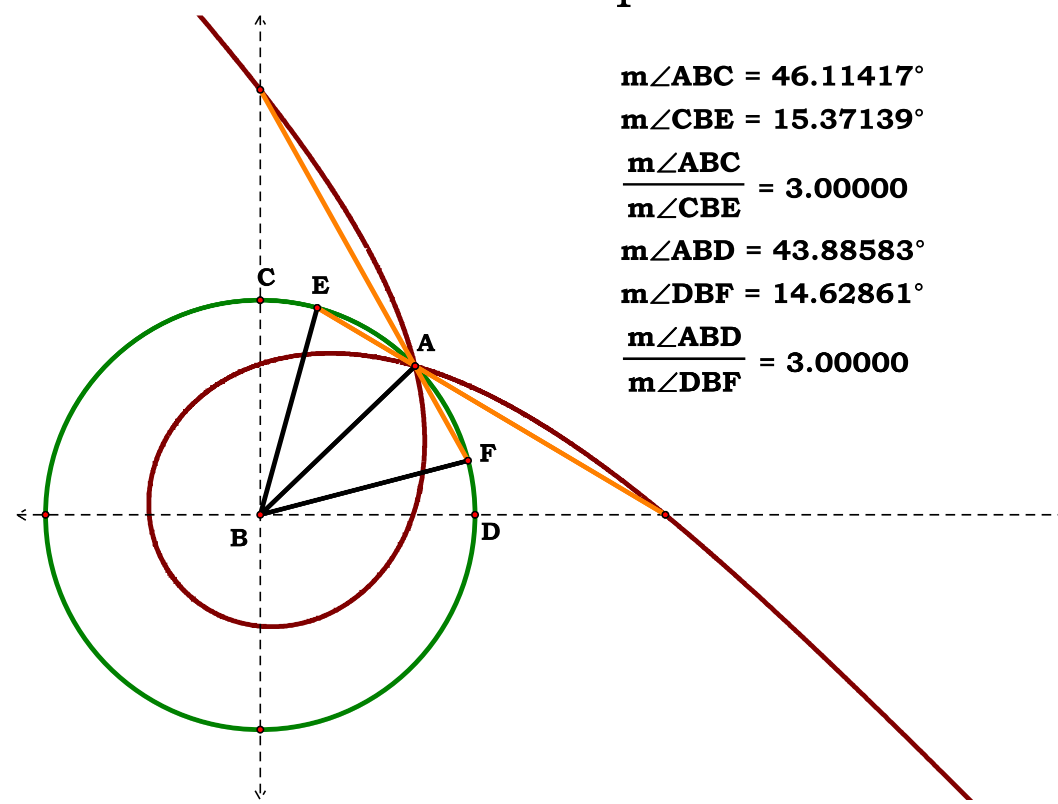
Construct a segment DE, where D is on the circle AB, and E on the circle BC.



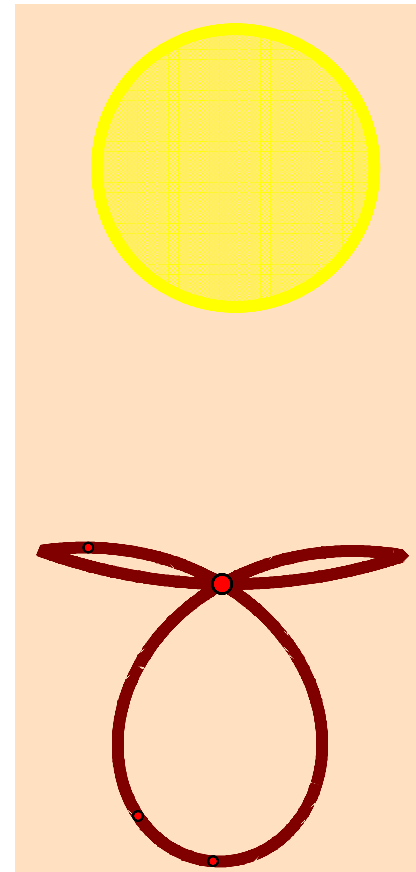
From the mid-point of DE, construct the perpendicular FG. At G through D and E, draw the third circle and denote its intersection with AB as H, and construct the Loci of H relative to B.



The end result can be viewed in Harvey's dressing room, to view Harvey and his magic ears. Keep in mind that one will have to redraw the intersection lines for the quadrant that A resides in.



So, if you make a mistake in the drawing, well, you just may end up with another brat fostered by Harvey, watching the sun.

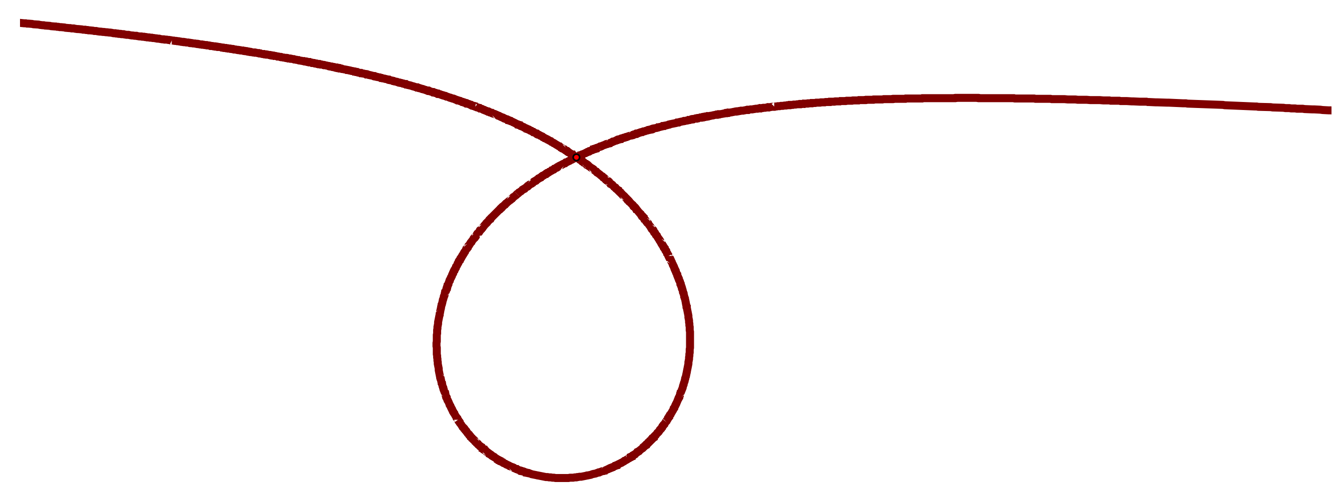


Harvey, The Full Service Rabbit

Sunday, June 18, 2023

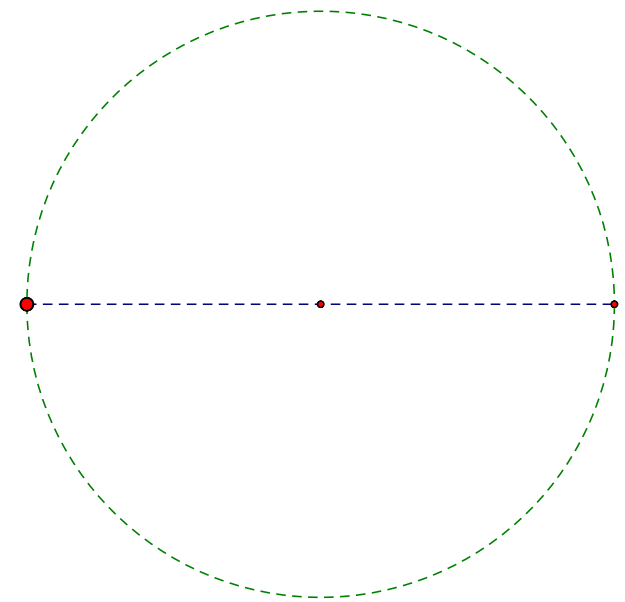
Back in ’06, I found Harvey. You might be familiar with Harvey, he made a feature film once with James Steward, by the same name. After Harvey retired, no one has seen him, or rather, since he was invisible, no one has ever seen him ’cept those who see pooka’s, ’cause Harvey is a pooka. Well, I ran into Harvey, as I said, and he tells me that he can trisect any angle using his head or his ears, whichever was required. I said to him, so what is that to me? If I cannot do it, then it is of no interest to me. I can teach you, said he, and so I learnt.

The following is how to draw Harvey’s head, and I got this straight from the rabbit himself.



That is exactly what the shape of Harvey's head is, and this is how to draw it, because, it can do what no human has been able to do, so we have to leave it up to the magic of a pooka.

One starts drawing Harvey's head with a perfect circle, like so.

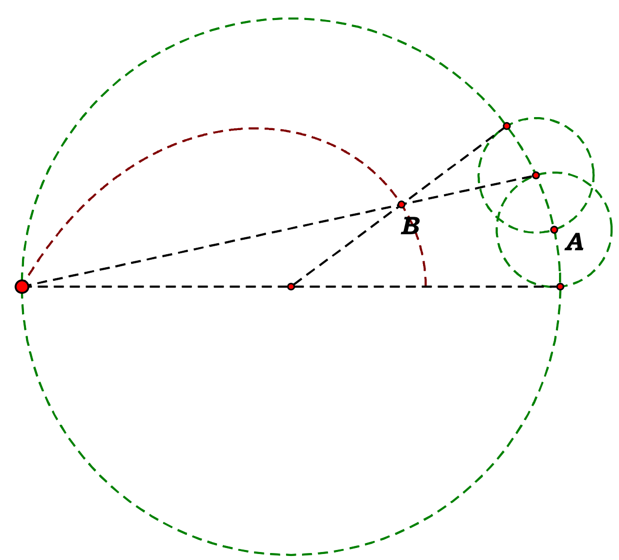


So, watch the two construction vids, and be warned, a point in any number of places can make a loci, and the figure can make many, but for now, we need only one method for this tool.

Trisection

Descriptions.

The most common sense method of finding the loci for angular division is very simple, B will always be proportional to A. So, let us write it up in a slightly different form.



$$AD := 1 \quad AB := \frac{AD}{2} \quad AC := .5833333$$

$$AM := 2 \cdot AD \quad CD := AD - AC \quad CJ := \sqrt{AC \cdot CD}$$

$$AJ := \sqrt{AC^2 + CJ^2} \quad JF := AJ \quad JK := AB \quad JG := \frac{AB^2}{JF}$$

$$AG := AJ - JG \quad AH := \frac{AC \cdot AG}{AJ} \quad AH = 0.333333 \quad GH := \frac{CJ \cdot AG}{AJ} \quad GH = 0.281718$$

Definitions.

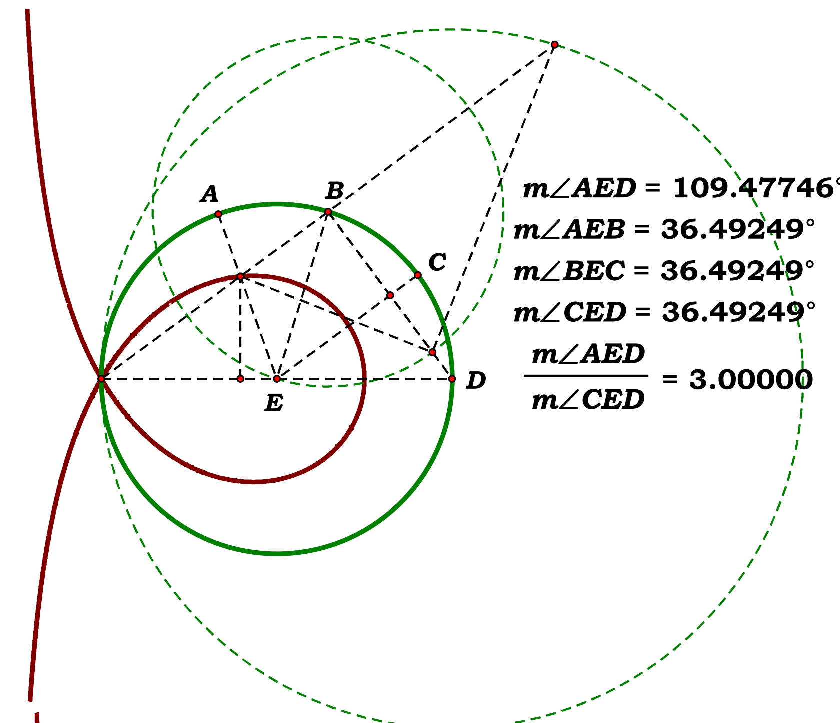
$$AD - 1 = 0 \quad AB - \frac{1}{2} = 0 \quad AC - AC = 0 \quad AM - 2 = 0 \quad CD - (1 - AC) = 0$$

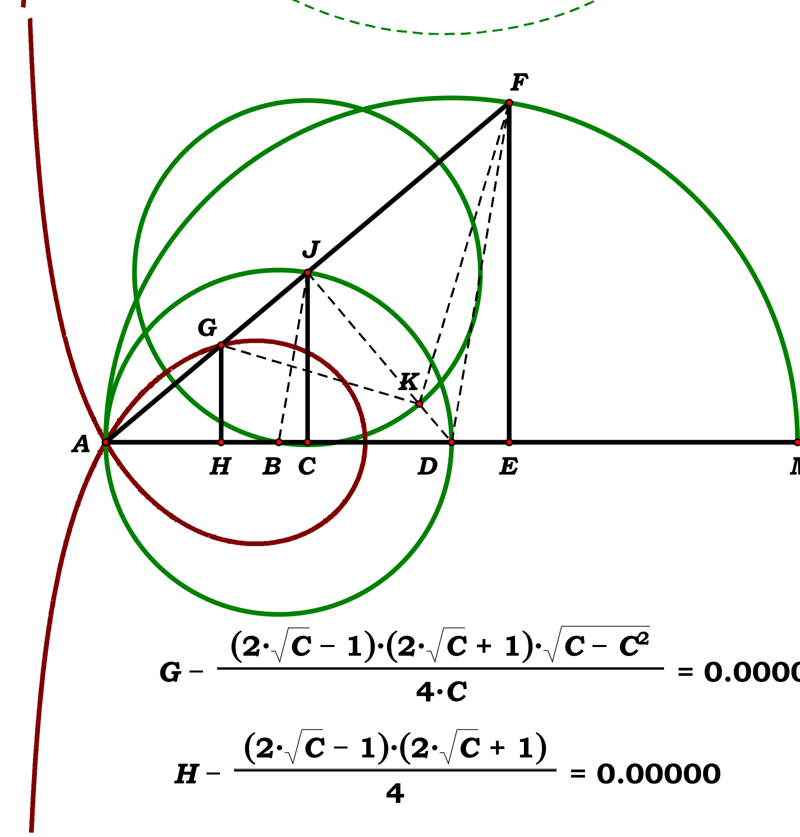
$$CJ - \sqrt{\left(AC - AC^2\right)} = 0 \quad AJ - \sqrt{AC} = 0 \quad JF - AJ = 0 \quad JK - AB = 0$$

$$JG - \frac{1}{4 \cdot \sqrt{AC}} = 0 \quad AG - \frac{\left(2 \cdot \sqrt{AC} - 1\right) \cdot \left(2 \cdot \sqrt{AC} + 1\right)}{4 \cdot \sqrt{AC}} = 0$$

$$GH - \frac{\left(2 \cdot \sqrt{AC} - 1\right) \cdot \left(2 \cdot \sqrt{AC} + 1\right) \cdot \sqrt{AC - AC^2}}{4 \cdot AC} = 0 \quad AH - \frac{\left(2 \cdot \sqrt{AC} - 1\right) \cdot \left(2 \cdot \sqrt{AC} + 1\right)}{4} = 0$$

Note from Euclid to Trigonometry: I'sm sorry, did I spoil your moment?





$A = 0.00000$
$H = 0.33333$
$B = 0.50000$
$C = 0.58333$
$D = 1.00000$
$E = 1.16667$
$N = 2.00000$
$G = 0.28172$
$J = 0.49301$
$F = 0.98601$

$$G - \frac{(2 \cdot \sqrt{C} - 1) \cdot (2 \cdot \sqrt{C} + 1) \cdot \sqrt{C - C^2}}{4 \cdot C} = 0.00000$$

$$H - \frac{(2 \cdot \sqrt{C} - 1) \cdot (2 \cdot \sqrt{C} + 1)}{4} = 0.00000$$

Whenever one has a fraction, say 7/1, 7/2 etc, instead of thinking something stupid, such as seven divided by one, say of a set of seven, this is the fist, and of a set of seven, this is the second.